Contents

CHAPTER

23

Springs

1. Introduction.
2. Types of Springs.
3. Material for Helical Springs.
4. Standard Size of Spring Wire.
5. Terms used in Compression Springs.
6. End Connections for Compression Helical Springs.
7. End Connections for Tension Helical Springs.
8. Stresses in Helical Springs of Circular Wire.
9. Deflection of Helical Springs of Circular Wire.
10. Eccentric Loading of Springs.
11. Buckling of Compression Springs.
12. Surge in Springs.
13. Energy Stored in Helical Springs of Circular Wire.
14. Stress and Deflection in Helical Springs of Non-circular Wire.
15. Helical Springs Subjected to Fatigue Loading.
16. Springs in Series.
17. Springs in Parallel.
18. Concentric or Composite Springs.
19. Helical Torsion Springs.
20. Flat Spiral Springs.
21. Leaf Springs.
22. Construction of Leaf Springs.
23. Equalised Stresses in Spring Leaves (Nipping).
24. Length of Leaf Spring Leaves.

23.1 Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows :

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring-loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

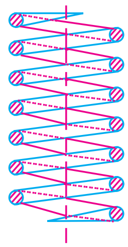
23.2 Types of Springs

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

Top

Contents

1. *Helical springs.* The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig. 23.1 (*a*) and ***tension helical spring*** as shown in Fig. 23.1 (*b*).



(*a*) Compression helical spring.

(*b*) Tension helical spring.

Fig. 23.1. Helical springs.

The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

In ***open coiled helical springs,*** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

(*a*) These are easy to manufacture.

(*b*) These are available in wide range.

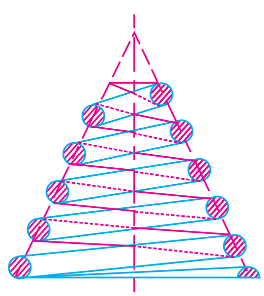
(*c*) These are reliable.

(*d*) These have constant spring rate.

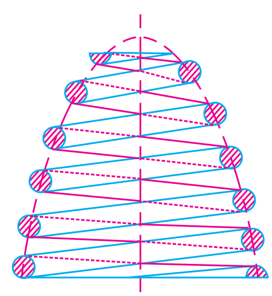
(*e*) Their performance can be predicted more accurately.

(*f*) Their characteristics can be varied by changing dimensions.

2. *Conical and volute springs.* The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. 23.2 (*a*), is wound with a uniform pitch whereas the volute springs, as shown in Fig. 23.2 (*b*), are wound in the form of paraboloid with constant pitch



(*a*) Conical spring.



(*b*) Volute spring.

Fig. 23.2. Conical and volute springs.

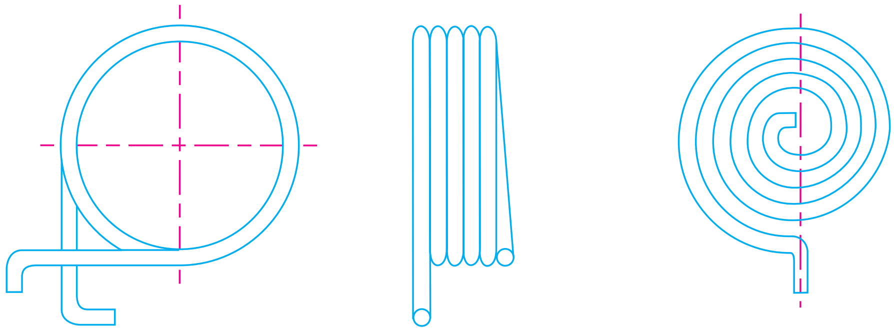
Top

and lead angles. The springs may be made either partially or completely telescoping. In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate. This characteristic is sometimes utilised in vibration problems where springs are used to support a body that has a varying mass.

The major stresses produced in conical and volute springs are also shear stresses due to twisting.

3. *Torsion springs.* These springs may be of ***helical*** or ***spiral*** type as shown in Fig. 23.3. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.

The major stresses produced in torsion springs are tensile and compressive due to bending.



(*a*) Helical torsion spring. (*b*) Spiral torsion spring.

Fig. 23.3. Torsion springs.

4. *Laminated or leaf springs.* The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. 23.4. These are mostly used in automobiles.

The major stresses produced in leaf springs are tensile and compressive stresses.

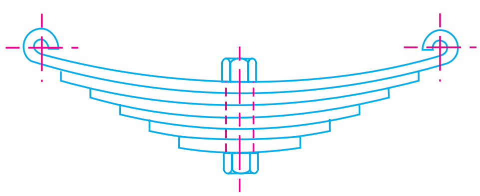


Fig. 23.4. Laminated or leaf springs.

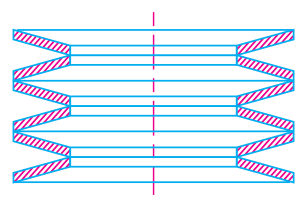


Fig. 23.5. Disc or bellevile springs.

5. *Disc or bellevile springs.* These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. 23.5. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

23.3 Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used *i.e.* severe service, average service or light service.

Severe service means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

Average service includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

Light service includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese. Music wire is used for small springs. Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.

Table 23.1 shows the values of allowable shear stress, modulus of rigidity and modulus of elasticity for various materials used for springs.

The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot. The strength of the wires varies with size, smaller size wires have greater strength and less ductility, due to the greater degree of cold working.

Table 23.1. Values of allowable shear stress, Modulus of elasticity and Modulus of rigidity for various spring materials.

Material	*Allowable shear stress (τ) MPa*			*Modulus of rigidity (G) kN/m²*	*Modulus of elasticity (E) kN/mm²*
	Severe service	*Average service*	*Light service*		
1. Carbon steel					
(*a*) Upto to 2.125 mm dia.	420	525	651	80	210
(*b*) 2.125 to 4.625 mm	385	483	595		
(*c*) 4.625 to 8.00 mm	336	420	525		
(*d*) 8.00 to 13.25 mm	294	364	455		
(*e*) 13.25 to 24.25 mm	252	315	392		
(*f*) 24.25 to 38.00 mm	224	280	350		
2. Music wire	392	490	612		
3. Oil tempered wire	336	420	525		
4. Hard-drawn spring wire	280	350	437.5		
5. Stainless-steel wire	280	350	437.5	70	196
6. Monel metal	196	245	306	44	105
7. Phosphor bronze	196	245	306	44	105
8. Brass	140	175	219	35	100

23.4 Standard Size of Spring Wire

The standard size of spring wire may be selected from the following table :

Table 23.2. Standard wire gauge (SWG) number and corresponding diameter of spring wire.

SWG	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*	*SWG*	*Diameter (mm)*
7/0	12.70	7	4.470	20	0.914	33	0.2540
6/0	11.785	8	4.064	21	0.813	34	0.2337
5/0	10.973	9	3.658	22	0.711	35	0.2134
4/0	10.160	10	3.251	23	0.610	36	0.1930
3/0	9.490	11	2.946	24	0.559	37	0.1727
2/0	8.839	12	2.642	25	0.508	38	0.1524
0	8.229	13	2.337	26	0.457	39	0.1321
1	7.620	14	2.032	27	0.4166	40	0.1219
2	7.010	15	1.829	28	0.3759	41	0.1118
3	6.401	16	1.626	29	0.3454	42	0.1016
4	5.893	17	1.422	30	0.3150	43	0.0914
5	5.385	18	1.219	31	0.2946	44	0.0813
6	4.877	19	1.016	32	0.2743	45	0.0711

23.5 Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. *Solid length.* When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be ***solid***. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$$L_S = n'.d$$

where n' = Total number of coils, and

d = Diameter of the wire.

2. *Free length.* The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

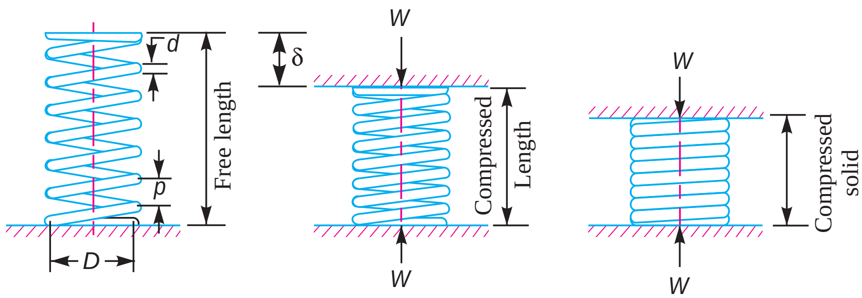


Fig. 23.6. Compression spring nomenclature.

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\,\delta_{max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. *Spring index.* The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. *Spring rate.* The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

* In actual practice, the compression springs are seldom designed to close up under the maximum working load and for this purpose a clearance (or clash allowance) is provided between the adjacent coils to prevent closing of the coils during service. It may be taken as 15 per cent of the maximum deflection.

Contents

5. *Pitch.* The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $$p = \frac{\text{Free length}}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $$p = \frac{L_F - L_S}{n'} + d$$

where L_F = Free length of the spring,

L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :

(*a*) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(*b*) The spring should not close up before the maximum service load is reached.

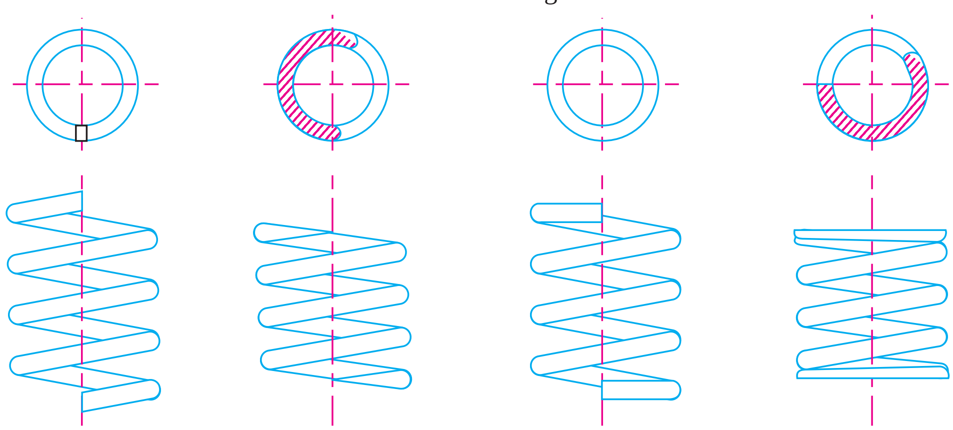
Note : In designing a tension spring (See Example 23.8), the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,

$$L_F = n.d + (n - 1)$$

and pitch of the coil, $$p = \frac{L_F}{n - 1}$$

23.6 End Connections for Compression Helical Springs

The end connections for compression helical springs are suitably formed in order to apply the load. Various forms of end connections are shown in Fig. 23.7.



(*a*) Plain ends. (*b*) Ground ends. (*c*) Squared ends. (*d*) Squared and ground ends.

Fig 23.7. End connections for compression helical spring.

In all springs, the end coils produce an eccentric application of the load, increasing the stress on one side of the spring. Under certain conditions, especially where the number of coils is small, this effect must be taken into account. The nearest approach to an axial load is secured by squared and ground ends, where the end turns are squared and then ground perpendicular to the helix axis. It may be noted that part of the coil which is in contact with the seat does not contribute to spring action and hence are termed as ***inactive coils***. The turns which impart spring action are known as ***active turns.*** As the load increases, the number of inactive coils also increases due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads. The following table shows the total number of turns, solid length and free length for different types of end connections.

Top

Table 23.3. Total number of turns, solid length and free length for different types of end connections.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n + 1)\, d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n + 2$	$(n + 3)\, d$	$p \times n + 3d$
4. Squared and ground ends	$n + 2$	$(n + 2)\, d$	$p \times n + 2d$

where n = Number of active turns,

p = Pitch of the coils, and

d = Diameter of the spring wire.

Tension helical spring

23.7 End Connections for Tension Helical Springs

The tensile springs are provided with hooks or loops as shown in Fig. 23.8. These loops may be made by turning whole coil or half of the coil. In a tension spring, large stress concentration is produced at the loop or other attaching device of tension spring.

The main disadvantage of tension spring is the failure of the spring when the wire breaks. A compression spring used for carrying a tensile load is shown in Fig. 23.9.

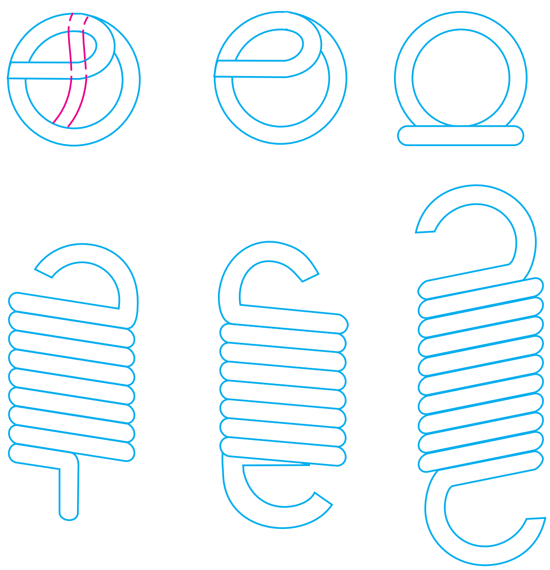


Fig. 23.8. End connection for tension helical springs.

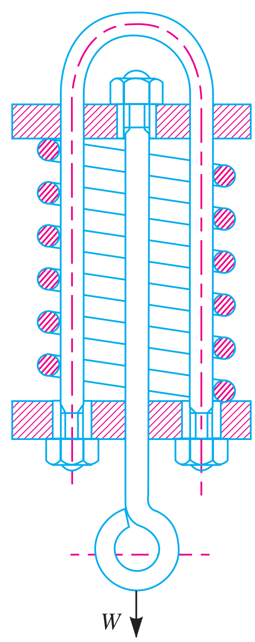


Fig. 23.9. Compression spring for carrying tensile load.

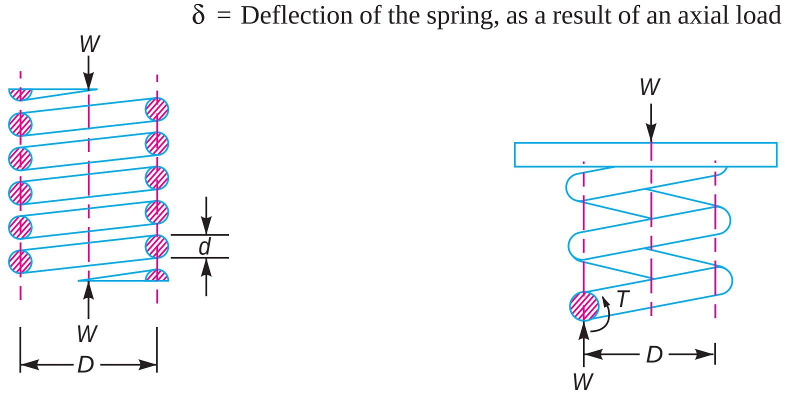
Note : The total number of turns of a tension helical spring must be equal to the number of turns (n) between the points where the loops start plus the equivalent turns for the loops. It has been found experimentally that half turn should be added for each loop. Thus for a spring having loops on both ends, the total number of active turns,

$$n' = n + 1$$

23.8 Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. 23.10 (a).

Let
D = Mean diameter of the spring coil,
d = Diameter of the spring wire,
n = Number of active coils,
G = Modulus of rigidity for the spring material,
W = Axial load on the spring,
τ = Maximum shear stress induced in the wire,
C = Spring index = D/d,
p = Pitch of the coils, and
δ = Deflection of the spring, as a result of an axial load W.



(a) Axially loaded helical spring.

(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Fig. 23.10

Now consider a part of the compression spring as shown in Fig. 23.10 (b). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig. 23.10 (b), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad \tau_1 = \frac{8 W.D}{\pi\, d^3} \qquad \ldots(i)$$

The torsional shear stress diagram is shown in Fig. 23.11 (a).

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

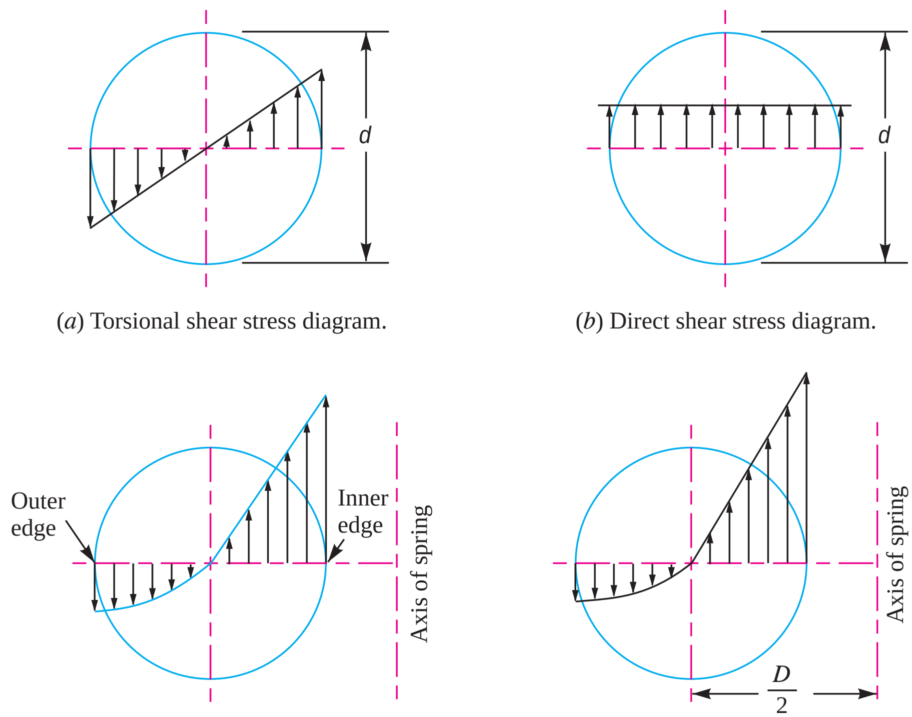
1. Direct shear stress due to the load W, and
2. Stress due to curvature of wire.

We know that direct shear stress due to the load W,

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi\, d^2} \quad \text{...}(ii)$$

The direct shear stress diagram is shown in Fig. 23.11 (*b*) and the resultant diagram of torsional shear stress and direct shear stress is shown in Fig. 23.11 (*c*).



(*a*) Torsional shear stress diagram.

(*b*) Direct shear stress diagram.

(*c*) Resultant torsional shear and direct shear stress diagram.

(*d*) Resultant torsional shear, direct shear and curvature shear stress diagram.

Fig. 23.11. Superposition of stresses in a helical spring.

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi\, d^3} \pm \frac{4W}{\pi\, d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outer edge of the wire. Since the stress is maximum at the inner edge of the wire, therefore

Maximum shear stress induced in the wire,

$$= \text{Torsional shear stress} + \text{Direct shear stress}$$

$$= \frac{8W.D}{\pi\, d^3} + \frac{4W}{\pi\, d^2} = \frac{8W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad \text{...(iii)}$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \dfrac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\dfrac{8\,WD}{\pi\,d^3} \times \dfrac{1}{2C}\right)$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation ***(iii)***. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 23.11 (*d*).

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad \text{...(iv)}$$

where $K = \dfrac{4C - 1}{4C - 4} + \dfrac{0.615}{C}$

The values of K for a given spring index (C) may be obtained from the graph as shown in Fig. 23.12.

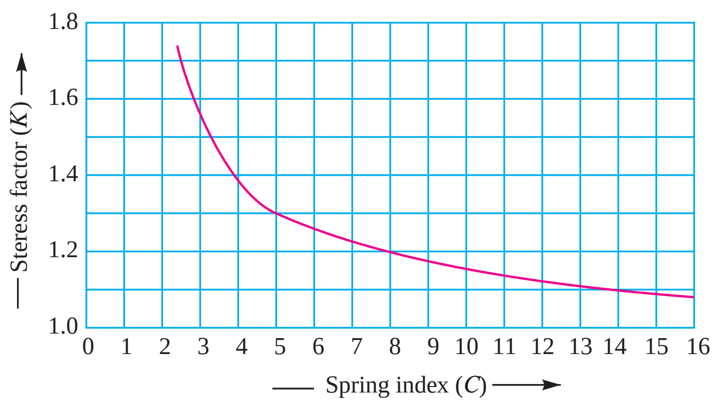


Fig. 23.12. Wahl's stress factor for helical springs.

We see from Fig. 23.12 that Wahl's stress factor increases very rapidly as the spring index decreases. The spring mostly used in machinery have spring index above 3.

Note: The Wahl's stress factor (K) may be considered as composed of two sub-factors, K_S and K_C, such that

$$K = K_S \times K_C$$

where K_S = Stress factor due to shear, and

K_C = Stress concentration factor due to curvature.

23.9 Deflection of Helical Springs of Circular Wire

In the previous article, we have discussed the maximum shear stress developed in the wire. We know that

Total active length of the wire,

$$l = \text{Length of one coil} \times \text{No. of active coils} = \pi\, D \times n$$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$\therefore$ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \text{...}(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \qquad \theta = \frac{T.l}{J.G} \qquad \text{...}\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$$= \frac{\pi}{32} \times d^4, \; d \text{ being the diameter of spring wire.}$$

and G = Modulus of rigidity for the material of the spring wire.

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\, D.n}{\frac{\pi}{32} \times d^4\, G} = \frac{16\, W.D^2.n}{G.d^4} \qquad \text{...}(ii)$$

Substituting this value of θ in equation (*i*), we have

$$\delta = \frac{16\, W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\, W.D^3.n}{G.d^4} = \frac{8\, W.C^3.n}{G.d} \qquad \text{... } (\because\; C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\, D^3.n} = \frac{G.d}{8\, C^3.n} = \text{constant}$$

23.10 Eccentric Loading of Springs

Sometimes, the load on the springs does not coincide with the axis of the spring, *i.e.* the spring is subjected to an eccentric load. In such cases, not only the safe load for the spring reduces, the stiffness of the spring is also affected. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance e from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor $\dfrac{D}{2\,e + D}$, where D is the mean diameter of the spring.

23.11 Buckling of Compression Springs

It has been found experimentally that when the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load as shown in Fig. 23.13. The critical axial load (W_{cr}) that causes buckling may be calculated by using the following relation, *i.e.*

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio L_F / D.

Contents

The buckling factor (K_B) for the hinged end and built-in end springs may be taken from the following table.

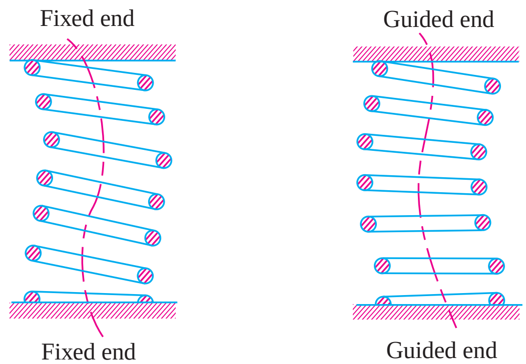


Fig. 23.13. Buckling of compression springs.

Table 23.4. Values of buckling factor (K_B).

L_F/D	*Hinged end spring*	*Built-in end spring*	L_F / D	*Hinged end spring*	*Built-in end spring*
1	0.72	0.72	5	0.11	0.53
2	0.63	0.71	6	0.07	0.38
3	0.38	0.68	7	0.05	0.26
4	0.20	0.63	8	0.04	0.19

It may be noted that a ***hinged end spring*** is one which is supported on pivots at both ends as in case of springs having plain ends where as a ***built-in end spring*** is one in which a squared and ground end spring is compressed between two rigid and parallel flat plates.

It order to avoid the buckling of spring, it is either mounted on a central rod or located on a tube. When the spring is located on a tube, the clearance between the tube walls and the spring should be kept as small as possible, but it must be sufficient to allow for increase in spring diameter during compression.

In railway coaches strongs springs are used for suspension.

Top

23.12 Surge in Springs

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge.***

It has been found that the natural frequency of spring should be atleast twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies upto twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2.n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.

2. By using springs of high natural frequency.

3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

Example 23.1. *A compression coil spring made of an alloy steel is having the following specifications :*

Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.

If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.

Solution. Given : D = 50 mm ; d = 5 mm ; *n = 20 ; W = 500 N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi\, d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$$= 534.7 \text{ MPa}$$ **Ans.**

* Superfluous data.

Top

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : d = 6 mm ; D_o = 75 mm ; τ = 350 MPa = 350 N/mm² ; G = 84 kN/mm² = 84 × 10³ N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index,

$$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\,W.D}{\pi\, d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$$\therefore \quad W = 350 / 0.848 = 412.7 \text{ N } \textbf{Ans.}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$$\therefore \quad W = 350 / 0.913 = 383.4 \text{ N } \textbf{Ans.}$$

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$

Example 23.3. *Design a spring for a balance to measure 0 to 1000 N over a scale of length 80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approximate number of turns is 30. The modulus of rigidity is 85 kN/mm². Also calculate the maximum shear stress induced.*

Solution. Given : W = 1000 N ; δ = 80 mm ; n = 30 ; G = 85 kN/mm² = 85 × 10³ N/mm²

Design of spring

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire, and

C = Spring index = D/d.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($D_o = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\,C^3}{85\,d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that d = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm } \textbf{Ans.}$$

Since the value of D_o = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, d = 4 mm is correct.

Maximum shear stress induced

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

∴ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$

$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa } \textbf{Ans.}$$

Example 23.4. *A mechanism used in printing machinery consists of a tension spring assembled with a preload of 30 N. The wire diameter of spring is 2 mm with a spring index of 6. The spring has 18 active coils. The spring wire is hard drawn and oil tempered having following material properties:*

Design shear stress = 680 MPa

Modulus of rigidity = 80 kN/mm²

Determine : 1. the initial torsional shear stress in the wire; 2. spring rate; and 3. the force to cause the body of the spring to its yield strength.

Tension springs are widely used in printing machines.

Solution. Given : W_i = 30 N ; d = 2 mm ; $C = D/d$ = 6 ; n = 18 ; τ = 680 MPa = 680 N/mm² ; G = 80 kN/mm² = 80×10^3 N/mm²

1. *Initial torsional shear stress in the wire*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

∴ Initial torsional shear stress in the wire,

$$\tau_i = K \times \frac{8\, W_i \times C}{\pi\, d^2} = 1.2525 \times \frac{8 \times 30 \times 6}{\pi \times 2^2} = 143.5 \text{ N/mm}^2$$

$$= 143.5 \text{ MPa } \textbf{Ans.}$$

2. *Spring rate*

We know that spring rate (or stiffness of the spring),

$$= \frac{G.d}{8\, C^3.n} = \frac{80 \times 10^3 \times 2}{8 \times 6^3 \times 18} = 5.144 \text{ N/mm } \textbf{Ans.}$$

3. *Force to cause the body of the spring to its yield strength*

Let W = Force to cause the body of the spring to its yield strength.

We know that design or maximum shear stress (τ),

$$680 = K \times \frac{8\, W.C}{\pi\, d^2} = 1.2525 \times \frac{8W \times 6}{\pi \times 2^2} = 4.78\, W$$

∴ $W = 680 / 4.78 = 142.25$ N **Ans.**

Example 23.5. *Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5.*

The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm².

Take Wahl's factor, $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$ *, where C = Spring index.*

Solution. Given : W = 1000 N ; δ = 25 mm ; $C = D/d = 5$; τ = 420 MPa = 420 N/mm² ; G = 84 kN/mm² = 84 × 10³ N/mm²

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\, W.C}{\pi\, d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\, d^2} = \frac{16\,677}{d^2}$$

∴ $d^2 = 16\,677 / 420 = 39.7$ or $d = 6.3$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

∴ Mean diameter of the spring coil,

$D = C.d = 5\,d = 5 \times 6.401 = 32.005$ mm **Ans.** ... ($\because C = D/d = 5$)

and outer diameter of the spring coil,

$D_o = D + d = 32.005 + 6.401 = 38.406$ mm **Ans.**

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\ (5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\ n$$

$\therefore$ $n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$n' = n + 2 = 14 + 2 = 16$ **Ans.**

3. *Free length of the spring*

We know that free length of the spring

$= n'.d + \delta + 0.15\ \delta = 16 \times 6.401 + 25 + 0.15 \times 25$

$= 131.2$ mm **Ans.**

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm} \quad \textbf{Ans.}$$

Example 23.6. *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm².*

Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$\therefore$ $d^2 = 6875 / 82.48 = 83.35$ or $d = 9.13$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$D = 5d = 5 \times 9.49 = 47.45$ mm **Ans.**

We know that outer diameter of the spring coil,

$D_o = D + d = 47.45 + 9.49 = 56.94$ mm **Ans.**

and inner diameter of the spring coil,

$D_i = D - d = 47.45 - 9.49 = 37.96$ mm **Ans.**

2. *Number of turns of the spring coil*

Let n = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for $W = 500$ N) is 6 mm.

Contents

We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

$\therefore$ $n = 6 / 0.63 = 9.5$ say 10 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \text{ Ans.}$$

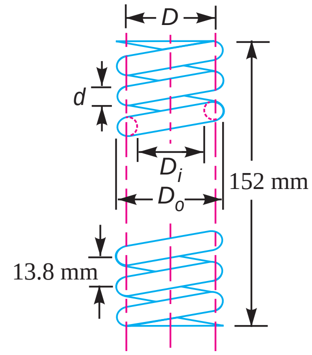


Fig. 23.14

3. Free length of the spring

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm Ans.}$$

4. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm Ans.}$$

The spring is shown in Fig. 23.14.

Example 23.7. *Design and draw a valve spring of a petrol engine for the following operating conditions :*

Spring load when the valve is open = 400 N
Spring load when the valve is closed = 250 N
Maximum inside diameter of spring = 25 mm
Length of the spring when the valve is open = 40 mm
Length of the spring when the valve is closed = 50 mm
Maximum permissible shear stress = 400 MPa

Solution. Given : $W_1 = 400$ N ; $W_2 = 250$ N ; $D_i = 25$ mm ; $l_1 = 40$ mm ; $l_2 = 50$ mm ; $\tau = 400$ MPa $= 400$ N/mm^2

Petrol engine.

1. Mean diameter of the spring coil

Let d = Diameter of the spring wire in mm, and

D = Mean diameter of the spring coil

= Inside dia. of spring + Dia. of spring wire = $(25 + d)$ mm

Since the diameter of the spring wire is obtained for the maximum spring load (W_1), therefore maximum twisting moment on the spring,

Top

$$T = W_1 \times \frac{D}{2} = 400\left(\frac{25+d}{2}\right) = (5000 + 200\,d)\text{ N-mm}$$

We know that maximum twisting moment (T),

$$(5000 + 200\,d) = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 400 \times d^3 = 78.55\,d^3$$

Solving this equation by hit and trial method, we find that d = 4.2 mm.

From Table 23.2, we find that standard size of wire is *SWG* 7 having d = 4.47 mm.

Now let us find the diameter of the spring wire by taking Wahl's stress factor (K) into consideration.

We know that spring index,

$$C = \frac{D}{d} = \frac{25 + 4.47}{4.47} = 6.6 \qquad \text{... } (\because D = 25 + d)$$

∴ Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6.6 - 1}{4 \times 6.6 - 4} + \frac{0.615}{6.6} = 1.227$$

We know that the maximum shear stress (τ),

$$400 = K \times \frac{8\,W_1 . C}{\pi\,d^2} = 1.227 \times \frac{8 \times 400 \times 6.6}{\pi\,d^2} = \frac{8248}{d^2}$$

$$\therefore \quad d^2 = 8248 / 400 = 20.62 \quad \text{or} \quad d = 4.54 \text{ mm}$$

Taking larger of the two values, we have

$$d = 4.54 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 6 having diameter (d) = 4.877 mm.

∴ Mean diameter of the spring coil

$$D = 25 + d = 25 + 4.877 = 29.877 \text{ mm}$$ **Ans.**

and outer diameter of the spring coil,

$$D_o = D + d = 29.877 + 4.877 = 34.754 \text{ mm}$$ **Ans.**

2. Number of turns of the coil

Let n = Number of active turns of the coil.

We are given that the compression of the spring caused by a load of ($W_1 - W_2$), *i.e.* 400 – 250 = 150 N is $l_2 - l_1$, *i.e.* 50 – 40 = 10 mm. In other words, the deflection (δ) of the spring is 10 mm for a load (W) of 150 N

We know that the deflection of the spring (δ),

$$10 = \frac{8\,W . D^3 . n}{G . d^4} = \frac{8 \times 150\,(29.877)^3\,n}{80 \times 10^3\,(4.877)^4} = 0.707\,n$$

... (Taking $G = 80 \times 10^3$ N/mm²)

$$\therefore \quad n = 10 / 0.707 = 14.2 \text{ say } 15$$ **Ans.**

Taking the ends of the springs as squared and ground, the total number of turns of the spring,

$$n' = 15 + 2 = 17$$ **Ans.**

3. Free length of the spring

Since the deflection for 150 N of load is 10 mm, therefore the maximum deflection for the maximum load of 400 N is

$$\delta_{max} = \frac{10}{150} \times 400 = 26.67 \text{ mm}$$

Top

An automobile suspension and shock-absorber. The two links with green ends are turnbuckles.

∴ Free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 17 \times 4.877 + 26.67 + 0.15 \times 26.67 = 113.58 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{113.58}{17 - 1} = 7.1 \text{ mm } \textbf{Ans.}$$

Example 23.8. *Design a helical spring for a spring loaded safety valve (Ramsbottom safety valve) for the following conditions :*

Diameter of valve seat = 65 mm ; Operating pressure = 0.7 N/mm²; Maximum pressure when the valve blows off freely = 0.75 N/mm²; Maximum lift of the valve when the pressure rises from 0.7 to 0.75 N/mm² = 3.5 mm ; Maximum allowable stress = 550 MPa ; Modulus of rigidity = 84 kN/mm²; Spring index = 6.

Draw a neat sketch of the free spring showing the main dimensions.

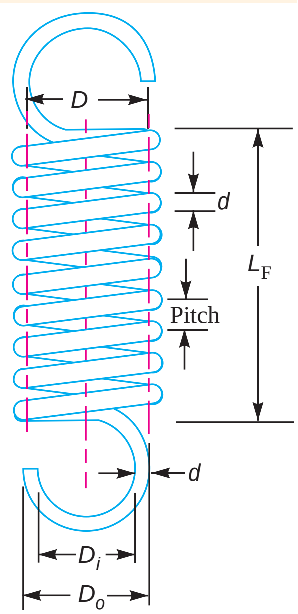


Fig. 23.15

Solution. Given : D_1 = 65 mm ; p_1 = 0.7 N/mm² ; p_2 = 0.75 N/mm² ; δ = 3.5 mm ; τ = 550 MPa = 550 N/mm² ; G = 84 kN/mm² = 84 × 10³ N/mm² ; C = 6

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

Since the safety valve is a Ramsbottom safety valve, therefore the spring will be under tension. We know that initial tensile force acting on the spring (*i.e.* before the valve lifts),

$$W_1 = \frac{\pi}{4}(D_1)^2\,p_1 = \frac{\pi}{4}(65)^2\,0.7 = 2323 \text{ N}$$

and maximum tensile force acting on the spring (*i.e.* when the valve blows off freely),

$$W_2 = \frac{\pi}{4}(D_1)^2\, p_2 = \frac{\pi}{4}(65)^2\, 0.75 = 2489 \text{ N}$$

∴ Force which produces the deflection of 3.5 mm,

$$W = W_2 - W_1 = 2489 - 2323 = 166 \text{ N}$$

Since the diameter of the spring wire is obtained for the maximum spring load (W_2), therefore maximum twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2489 \times \frac{6\,d}{2} = 7467\,d \qquad \ldots (\because C = D/d = 6)$$

We know that maximum twisting moment (T),

$$7467\,d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 550 \times d^3 = 108\,d^3$$

$$\therefore \quad d^2 = 7467 / 108 = 69.14 \quad \text{or} \quad d = 8.3 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 2/0 having diameter (d) = 8.839 mm **Ans.**

∴ Mean diameter of the coil,

$$D = 6\,d = 6 \times 8.839 = 53.034 \text{ mm} \quad \textbf{Ans.}$$

Outside diameter of the coil,

$$D_o = D + d = 53.034 + 8.839 = 61.873 \text{ mm} \quad \textbf{Ans.}$$

and inside diameter of the coil,

$$D_i = D - d = 53.034 - 8.839 = 44.195 \text{ mm} \quad \textbf{Ans.}$$

2. Number of turns of the coil

Let n = Number of active turns of the coil.

We know that the deflection of the spring (δ),

$$3.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 166 \times 6^3 \times n}{84 \times 10^3 \times 8.839} = 0.386\,n$$

$$\therefore \quad n = 3.5 / 0.386 = 9.06 \text{ say } 10 \quad \textbf{Ans.}$$

For a spring having loop on both ends, the total number of turns,

$$n' = n + 1 = 10 + 1 = 11 \quad \textbf{Ans.}$$

3. Free length of the spring

Taking the least gap between the adjacent coils as 1 mm when the spring is in free state, the free length of the tension spring,

$$L_F = n.d + (n - 1)\,1 = 10 \times 8.839 + (10 - 1)\,1 = 97.39 \text{ mm} \quad \textbf{Ans.}$$

4. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n - 1} = \frac{97.39}{10 - 1} = 10.82 \text{ mm} \quad \textbf{Ans.}$$

The tension spring is shown in Fig. 23.15.

Example 23.9. *A safety valve of 60 mm diameter is to blow off at a pressure of 1.2 N/mm². It is held on its seat by a close coiled helical spring. The maximum lift of the valve is 10 mm. Design a suitable compression spring of spring index 5 and providing an initial compression of 35 mm. The maximum shear stress in the material of the wire is limited to 500 MPa. The modulus of rigidity for the spring material is 80 kN/mm². Calculate : 1. Diameter of the spring wire, 2. Mean coil diameter, 3. Number of active turns, and 4. Pitch of the coil.*

Take Wahl's factor, $K = \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, *where C is the spring index.*

Solution. Given : Valve dia. = 60 mm ; Max. pressure = 1.2 N/mm^2 ; δ_2 = 10 mm ; C = 5 ; δ_1 = 35 mm ; τ = 500 MPa = 500 N/mm^2 ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. ***Diameter of the spring wire***

Let d = Diameter of the spring wire.

We know that the maximum load acting on the valve when it just begins to blow off,

$$W_1 = \text{Area of the valve} \times \text{Max. pressure}$$

$$= \frac{\pi}{4}(60)^2\,1.2 = 3394 \text{ N}$$

and maximum compression of the spring,

$$\delta_{max} = \delta_1 + \delta_2 = 35 + 10 = 45 \text{ mm}$$

Since a load of 3394 N keeps the valve on its seat by providing initial compression of 35 mm, therefore the maximum load on the spring when the valve is oepn (*i.e.* for maximum compression of 45 mm),

$$W = \frac{3394}{35} \times 45 = 4364 \text{ N}$$

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5} = 1.31$$

We also know that the maximum shear stress (τ),

$$500 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.31 \times \frac{8\times4364\times5}{\pi\,d^2} = \frac{72\,780}{d^2}$$

$\therefore$ d^2 = 72 780 / 500 = 145.6 or d = 12.06 mm

From Table 23.2, we shall take a standard wire of size *SWG* 7/0 having diameter (d) = 12.7 mm. **Ans.**

2. ***Mean coil diameter***

Let D = Mean coil diameter.

We know that the spring index,

$$C = D/d \quad \text{or} \quad D = C.d = 5 \times 12.7 = 63.5 \text{ mm } \textbf{Ans.}$$

3. ***Number of active turns***

Let n = Number of active turns.

We know that the maximum compression of the spring (δ),

$$45 = \frac{8\,W.C^3.n}{G.d} = \frac{8\times4364\times5^3\times n}{80\times10^3\times12.7} = 4.3\,n$$

$\therefore$ n = 45 / 4.3 = 10.5 say 11 **Ans.**

Taking the ends of the coil as squared and ground, the total number of turns,

$$n' = n + 2 = 11 + 2 = 13 \textbf{ Ans.}$$

Note : The valve of n may also be calculated by using

$$\delta_1 = \frac{8\,W_1.C^3.n}{G.d}$$

$$35 = \frac{8\times3394\times5^3\times n}{80\times10^3\times12.7} = 3.34\,n \quad \text{or} \quad n = 35/3.34 = 10.5 \text{ say } 11$$

4. Pitch of the coil

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max} = 13 \times 12.7 + 45 + 0.15 \times 45$$
$$= 216.85 \text{ mm } \textbf{Ans}$$

$$\therefore \quad \text{Pitch of the coil} = \frac{\text{Free length}}{n' - 1} = \frac{216.85}{13 - 1} = 18.1 \text{ mm } \textbf{Ans.}$$

Example 23.10. *In a spring loaded governor as shown in Fig. 23.16, the balls are attached to the vertical arms of the bell crank lever, the horizontal arms of which lift the sleeve against the pressure exerted by a spring. The mass of each ball is 2.97 kg and the lengths of the vertical and horizontal arms of the bell crank lever are 150 mm and 112.5 mm respectively. The extreme radii of rotation of the balls are 100 mm and 150 mm and the governor sleeve begins to lift at 240 r.p.m. and reaches the highest position with a 7.5 percent increase of speed when effects of friction are neglected. Design a suitable close coiled round section spring for the governor.*

Assume permissible stress in spring steel as 420 MPa, modulus of rigidity 84 kN/mm² and spring index 8. Allowance must be made for stress concentration, factor of which is given by

$$\frac{4C - 1}{4C - 4} + \frac{0.615}{C}, \text{ where } C \text{ is the spring index.}$$

Solution. Given : $m = 2.97$ kg ; $x = 150$ mm $= 0.15$ m ; $y = 112.5$ mm $= 0.1125$ m ; $r_2 = 100$ mm $= 0.1$ m ; $r_1 = 150$ mm $= 0.15$ m ; $N_2 = 240$ r.p.m. ; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm² ; $C = 8$

The spring loaded governor, as shown in Fig. 23.16, is a *Hartnell type governor. First of all, let us find the compression of the spring.

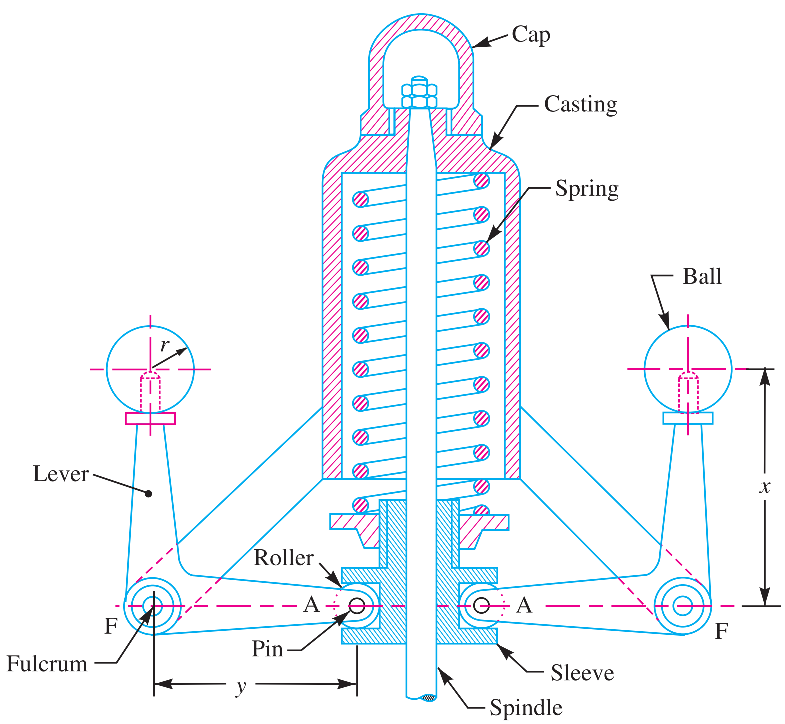


Fig. 23.16

* For further details, see authors' popular book on **'Theory of Machines'.**

Contents

We know that minimum angular speed at which the governor sleeve begins to lift,

$$\omega_2 = \frac{2\,\pi\,N_2}{60} = \frac{2\,\pi \times 240}{60} = 25.14 \text{ rad/s}$$

Since the increase in speed is 7.5%, therefore maximum speed,

$$\omega_1 = \omega_2 + \frac{7.5}{100} \times \omega_2 = 25.14 + \frac{7.5}{100} \times 25.14 = 27 \text{ rad/s}$$

The position of the balls and the lever arms at the maximum and minimum speeds is shown in Fig. 23.17 (*a*) and (*b*) respectively.

Let F_{C1} = Centrifugal force at the maximum speed, and

F_{C2} = Centrifugal force at the minimum speed.

We know that the spring force at the maximum speed (ω_1),

$$S_1 = 2\,F_{C1} \times \frac{x}{y} = 2\,m\,(\omega_1)^2\,r_1 \times \frac{x}{y} = 2 \times 2.97\,(27)^2\;0.15 \times \frac{0.15}{0.1125} = 866 \text{ N}$$

Similarly, the spring force at the minimum speed ω_2,

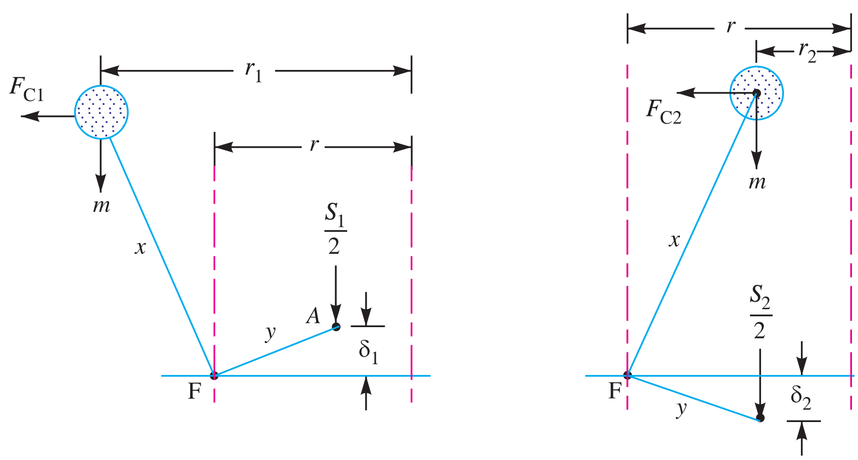
$$S_2 = 2\,F_{C2} \times \frac{x}{y} = 2m\,(\omega_2)^2\,r_2 \times \frac{x}{y} = 2 \times 2.97\,(25.14)^2\;0.1 \times \frac{0.15}{0.1125} = 500 \text{ N}$$

Since the compression of the spring will be equal to the lift of the sleeve, therefore compression of the spring,

$$\delta = \delta_1 + \delta_2 = (r_1 - r)\,\frac{y}{x} + (r - r_2)\,\frac{y}{x} = (r_1 - r_2)\,\frac{y}{x}$$

$$= (0.15 - 0.1)\,\frac{0.1125}{0.15} = 0.0375 \text{ m} = 37.5 \text{ mm}$$

This compression of the spring is due to the spring force of $(S_1 - S_2)$ *i.e.* (866 – 500) = 366 N.



(*a*) Maximum position. (*b*) Minimum position.

Fig. 23.17

1. *Diameter of the spring wire*

Let d = Diameter of the spring wire in mm.

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

Top

We also know that maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\,d^2} = 1.184 \times \frac{8 \times 866 \times 8}{\pi\,d^2} = \frac{20\ 885}{d^2}$$

... (Substituting $W = S_1$, the maximum spring force)

$\therefore$ $$d^2 = 20\ 885 / 420 = 49.7 \quad \text{or} \quad d = 7.05 \text{ mm}$$

From Table 23.2, we shall take the standard wire of size *SWG* 1 having diameter (d) = 7.62 mm **Ans.**

2. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil.

We know that the spring index,

$$C = D/d \quad \text{or} \quad D = C.d = 8 \times 7.62 = 60.96 \text{ mm } \textbf{Ans.}$$

3. *Number of turns of the coil*

Let n = Number of active turns of the coil.

We know that compression of the spring (δ),

$$37.5 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 366 \times 8^3 \times n}{84 \times 10^3 \times 7.62} = 2.34\,n$$

... (Substituting $W = S_1 - S_2$)

$\therefore$ $$n = 37.5 / 2.34 = 16 \textbf{ Ans.}$$

and total number of turns using squared and ground ends,

$$n' = n + 2 = 16 + 2 = 18$$

4. *Free length of the coil*

Since the compression produced under a force of 366 N is 37.5 mm, therefore maximum compression produced under the maximum load of 866 N is,

$$\delta_{max} = \frac{37.5}{366} \times 866 = 88.73 \text{ mm}$$

We know that free length of the coil,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 18 \times 7.62 + 88.73 + 0.15 \times 88.73 = 239.2 \text{ mm } \textbf{Ans.}$$

5. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{239.2}{18 - 1} = 14.07 \text{ mm } \textbf{Ans.}$$

Example 23.11. *A single plate clutch is to be designed for a vehicle. Both sides of the plate are to be effective. The clutch transmits 30 kW at a speed of 3000 r.p.m. and should cater for an over load of 20%. The intensity of pressure on the friction surface should not exceed 0.085 N/mm^2 and the surface speed at the mean radius should be limited to 2300 m / min. The outside diameter of the surfaces may be assumed as 1.3 times the inside diameter and the coefficient of friction for the surfaces may be taken as 0.3. If the axial thrust is to be provided by six springs of about 25 mm mean coil diameter, design the springs selecting wire from the following gauges :*

SWG	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
Dia. (mm)	*5.893*	*5.385*	*4.877*	*4.470*	*4.064*	*3.658*	*3.251*	*2.946*	*2.642*

Safe shear stress is limited to 420 MPa and modulus of rigidity is 84 kN/mm^2.

Solution. Given : P = 30 kW = 30 × 10^3 W ; N = 3000 r.p.m. ; p = 0.085 N/mm^2 ; v = 2300 m/min ; $d_1 = 1.3\,d_2$ or $r_1 = 1.3\,r_2$; μ = 0.3 ; No. of springs = 6 ; D = 25 mm ; τ = 420 MPa = 420 N/mm^2 ; G = 84 kN/mm^2 = 84 × 10^3 N/mm^2

Contents

First of all, let us find the maximum load on each spring. We know that the mean torque transmitted by the clutch,

$$T_{mean} = \frac{P \times 60}{2\,\pi\,N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 3000} = 95.5 \text{ N-m}$$

Since an overload of 20% is allowed, therefore maximum torque to which the clutch should be designed is given by

$$T_{max} = 1.2\,T_{mean} = 1.2 \times 95.5 = 114.6 \text{ N-m} = 114\,600 \text{ N-mm} \quad ...(i)$$

Let r_1 and r_2 be the outside and inside radii of the friction surfaces. Since maximum intensity of pressure is at the inner radius, therefore for uniform wear,

$$^*p \times r_2 = C \text{ (a constant) or } C = 0.085\,r_2$$

We know that the axial thrust transmitted,

$$W = C \times 2\pi\,(r_1 - r_2) \quad ...(ii)$$

Since both sides of the plate are effective, therefore maximum torque transmitted,

$$T_{max} = \tfrac{1}{2}\,\mu \times W\,(r_1 + r_2)\,2 = 2\pi\,\mu.C\,[(r_1)^2 - (r_2)^2] \quad \text{... [From equation } (ii)]$$

$$114\,600 = 2\pi \times 0.3 \times 0.085\,r_2\,[(1.3\,r_2)^2 - (r_2)^2] = 0.11\,(r_2)^3$$

$\therefore$ $$(r_2)^3 = 114\,600 / 0.11 = 1.04 \times 10^6 \text{ or } r_2 = 101.4 \text{ say } 102 \text{ mm}$$

and $$r_1 = 1.3\,r_2 = 1.3 \times 102 = 132.6 \text{ say } 133 \text{ mm}$$

$\therefore$ Mean radius,

$$r = \frac{r_1 + r_2}{2} = \frac{133 + 102}{2} = 117.5 \text{ mm} = 0.1175 \text{ m}$$

We know that surface speed at the mean radius,

$$v = 2\,\pi\,r\,N = 2\,\pi \times 0.1175 \times 3000 = 2215 \text{ m/min}$$

Since the surface speed as obtained above is less than the permissible value of 2300 m/min, therefore the radii of the friction surface are safe.

We know that axial thrust,

$$W = C \times 2\pi\,(r_1 - r_2) = 0.085\,r_2 \times 2\pi\,(r_1 - r_2) \quad \text{... } (\because C = 0.085\,r_2)$$

$$= 0.085 \times 102 \times 2\pi\,(133 - 102) = 1689 \text{ N}$$

Since this axial thrust is to be provided by six springs, therefore maximum load on each spring,

$$W_1 = \frac{1689}{6} = 281.5 \text{ N}$$

1. *Diameter of the spring wire*

Let d = Diameter of the spring wire.

We know that the maximum torque transmitted,

$$T = W_1 \times \frac{D}{2} = 281.5 \times \frac{25}{2} = 3518.75 \text{ N-mm}$$

We also know that the maximum torque transmitted (T),

$$3518.75 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\,d^3$$

$\therefore$ $$d^3 = 3518.75 / 82.48 = 42.66 \text{ or } d = 3.494 \text{ mm}$$

Let us now find out the diameter of the spring wire by taking the stress factor (K) into consideration.

We know that the spring index,

$$C = \frac{D}{d} = \frac{25}{3.494} = 7.155$$

* Please refer Chapter 24 on Clutches.

Top

and Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 7.155 - 1}{4 \times 7.155 - 4} + \frac{0.615}{7.155} = 1.21$$

We know that the maximum shear stress (τ),

$$420 = K \times \frac{8\,W_1 . D}{\pi\, d^3} = 1.21 \times \frac{8 \times 281.5 \times 25}{\pi\, d^3} = \frac{21\,681}{d^3}$$

$\therefore$ $d^3 = 21\,681 / 420 = 51.6$ or $d = 3.72$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 8 having diameter (d) = 4.064 mm. **Ans.**

Outer diameter of the spring,

$$D_o = D + d = 25 + 4.064 = 29.064 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring,

$$D_i = D - d = 25 - 4.064 = 20.936 \text{ mm } \textbf{Ans.}$$

2. Free length of the spring

Let us assume the active number of coils (n) = 8. Therefore compression produced by an axial thrust of 281.5 N per spring,

$$\delta = \frac{8\,W_1 . D^3 . n}{G . d^4} = \frac{8 \times 281.5\,(25)^3\,8}{84 \times 10^3\,(4.064)^4} = 12.285 \text{ mm}$$

For square and ground ends, the total number of turns of the coil,

$$n' = n + 2 = 8 + 2 = 10$$

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 10 \times 4.064 + 12.285 + 0.15 \times 12.285 \text{ mm}$$
$$= 54.77 \text{ mm } \textbf{Ans.}$$

3. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{54.77}{10 - 1} = 6.08 \text{ mm } \textbf{Ans.}$$

23.13 Energy Stored in Helical Springs of Circular Wire

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W . \delta \qquad \ldots(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W . D}{\pi\, d^3} \text{ or } W = \frac{\pi\, d^3 . \tau}{8\,K . D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W . D^3 . n}{G . d^4} = \frac{8 \times \pi\, d^3 . \tau}{8\,K . D} \times \frac{D^3 . n}{G . d^4} = \frac{\pi\, \tau . D^2 . n}{K . d . G}$$

Substituting the values of W and δ in equation **(*i*)**, we have

$$U = \frac{1}{2} \times \frac{\pi d^3.\tau}{8 K.D} \times \frac{\pi \tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4 K^2.G} (\pi D.n) \left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4 K^2.G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi D.n) \left(\frac{\pi}{4} \times d^2\right)$$

Note : When a load (say P) falls on a spring through a height h, then the energy absorbed in a spring is given by

$$U = P(h + \delta) = \frac{1}{2} W.\delta$$

where W = Equivalent static load *i.e.* the gradually applied load which shall produce the same effect as by the falling load P, and

δ = Deflection produced in the spring.

Another view of an automobile shock-absorber

Example 23.12. *Find the maximum shear stress and deflection induced in a helical spring of the following specifications, if it has to absorb 1000 N-m of energy.*

Mean diameter of spring = 100 mm ; Diameter of steel wire, used for making the spring = 20 mm; Number of coils = 30 ; Modulus of rigidity of steel = 85 kN/mm^2.

Solution. Given : U = 1000 N-m ; D = 100 mm = 0.1 m ; d = 20 mm = 0.02 m ; n = 30 ; G = 85 kN/mm^2 = 85 × 10^9 N/m^2

Maximum shear stress induced

Let τ = Maximum shear stress induced.

We know that spring index,

$$C = \frac{D}{d} = \frac{0.1}{0.02} = 5$$

∴ Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

Volume of spring wire,

$$V = (\pi\, D.n)\left(\frac{\pi}{4} \times d^2\right) = (\pi \times 0.1 \times 30)\left[\frac{\pi}{4}\,(0.02)^2\right] \text{m}^3$$

$$= 0.002\,96 \text{ m}^3$$

We know that energy absorbed in the spring (U),

$$1000 = \frac{\tau^2}{4K^2.G} \times V = \frac{\tau^2}{4\,(1.31)^2\; 85 \times 10^9} \times 0.00296 = \frac{5\,\tau^2}{10^{15}}$$

$$\therefore \quad \tau^2 = 1000 \times 10^{15} / 5 = 200 \times 10^{15}$$

or $\tau = 447.2 \times 10^6 \text{ N/m}^2 = 447.2$ MPa **Ans.**

Deflection produced in the spring

We know that deflection produced in the spring,

$$\delta = \frac{\pi\tau.D^2\, n}{K.d.G} = \frac{\pi \times 447.2 \times 10^6\;(0.1)^2\;30}{1.31 \times 0.02 \times 85 \times 10^9} = 0.1893 \text{ m}$$

$$= 189.3 \text{ mm}$$ **Ans.**

Example 23.13. *A closely coiled helical spring is made of 10 mm diameter steel wire, the coil consisting of 10 complete turns with a mean diameter of 120 mm. The spring carries an axial pull of 200 N. Determine the shear stress induced in the spring neglecting the effect of stress concentration. Determine also the deflection in the spring, its stiffness and strain energy stored by it if the modulus of rigidity of the material is 80 kN/mm².*

Solution. Given : $d = 10$ mm ; $n = 10$; $D = 120$ mm ; $W = 200$ N ; $G = 80 \text{ kN/mm}^2 = 80 \times 10^3 \text{ N/mm}^2$

Shear stress induced in the spring neglecting the effect of stress concentration

We know that shear stress induced in the spring neglecting the effect of stress concentration is,

$$\tau = \frac{8\,W.D}{\pi\, d^3}\left(1 + \frac{d}{2D}\right) = \frac{8 \times 200 \times 120}{\pi\,(10)^3}\left[1 + \frac{10}{2 \times 120}\right] \text{N/mm}^2$$

$$= 61.1 \times 1.04 = 63.54 \text{ N/mm}^2 = 63.54 \text{ MPa}$$ **Ans.**

Deflection in the spring

We know that deflection in the spring,

$$\delta = \frac{8\,W.D^3 n}{G.d^4} = \frac{8 \times 200\;(120)^3\;10}{80 \times 10^3\;(10)^4} = 34.56 \text{ mm}$$ **Ans.**

Stiffness of the spring

We know that stiffness of the spring

$$= \frac{W}{\delta} = \frac{200}{34.56} = 5.8 \text{ N/mm}$$

Strain energy stored in the spring

We know that strain energy stored in the spring,

$$U = \frac{1}{2}\,W.\delta = \frac{1}{2} \times 200 \times 34.56 = 3456 \text{ N-mm} = 3.456 \text{ N-m}$$ **Ans.**

Example 23.14. *At the bottom of a mine shaft, a group of 10 identical close coiled helical springs are set in parallel to absorb the shock caused by the falling of the cage in case of a failure. The loaded cage weighs 75 kN, while the counter weight has a weight of 15 kN. If the loaded cage falls through a height of 50 metres from rest, find the maximum stress induced in each spring if it is made of 50 mm diameter steel rod. The spring index is 6 and the number of active turns in each spring is 20. Modulus of rigidity, G = 80 kN/mm².*

Top

Solution. Given : No. of springs = 10 ; W_1 = 75 kN = 75 000 N ; W_2 = 15 kN = 15 000 N ; h = 50 m = 50 000 mm ; d = 50 mm ; C = 6 ; n = 20 ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

We know that net weight of the falling load,

$$P = W_1 - W_2 = 75\,000 - 15\,000 = 60\,000 \text{ N}$$

Let W = The equivalent static (or gradually applied) load on each spring which can produce the same effect as by the falling load P.

We know that compression produced in each spring,

$$\delta = \frac{8\,W.C^3.n}{G.d} = \frac{8W \times 6^3 \times 20}{80 \times 10^3 \times 50} = 0.008\,64\,W \text{ mm}$$

Since the work done by the falling load is equal to the energy stored in the helical springs which are 10 in number, therefore,

$$P\,(h + \delta) = \frac{1}{2}\,W \times \delta \times 10$$

$$60\,000\,(50\,000 + 0.008\,64\,W) = \frac{1}{2}\,W \times 0.008\,64W \times 10$$

$$3 \times 10^9 + 518.4\,W = 0.0432\,W^2$$

or $\quad W^2 - 12\,000\,W - 69.4 \times 10^9 = 0$

$$\therefore \quad W = \frac{12\,000 \pm \sqrt{(12\,000)^2 + 4 \times 1 \times 69.4 \times 10^9}}{2} = \frac{12\,000 \pm 527\,000}{2}$$

$$= 269\,500 \text{ N} \qquad \text{... (Taking +ve sign)}$$

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.25$$

and maximum stress induced in each spring,

$$\tau = K \times \frac{8W.C}{\pi\,d^2} = 1.25 \times \frac{8 \times 269\,500 \times 6}{\pi\,(50)^2} = 2058.6 \text{ N/mm}^2$$

$$= 2058.6 \text{ MPa}$$ **Ans.**

Example 23.15. *A rail wagon of mass 20 tonnes is moving with a velocity of 2 m/s. It is brought to rest by two buffers with springs of 300 mm diameter. The maximum deflection of springs is 250 mm. The allowable shear stress in the spring material is 600 MPa. Design the spring for the buffers.*

Buffers have springs inside to absorb shock.

Solution. Given : m = 20 t = 20 000 kg ; v = 2 m/s ; D = 300 mm ; δ = 250 mm ; τ = 600 MPa = 600 N/mm^2

1. Diameter of the spring wire

Let d = Diameter of the spring wire.

We know that kinetic energy of the wagon

$$= \frac{1}{2}\,m\,v^2 = \frac{1}{2} \times 20\,000\,(2)^2 = 40\,000 \text{ N-m} = 40 \times 10^6 \text{ N-mm} \qquad ...(i)$$

Let W be the equivalent load which when applied gradually on each spring causes a deflection of 250 mm. Since there are two springs, therefore

Energy stored in the springs

$$= \frac{1}{2} \times W.\delta \times 2 = W.\delta = W \times 250 = 250\,W \text{ N-mm} \quad ...(ii)$$

From equations (*i*) and (*ii*), we have

$\therefore \quad W = 40 \times 10^6 / 250 = 160 \times 10^3$ N

We know that torque transmitted by the spring,

$$T = W \times \frac{D}{2} = 160 \times 10^3 \times \frac{300}{2} = 24 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the spring (T),

$$24 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 600 \times d^3 = 117.8\ d^3$$

$\therefore \quad d^3 = 24 \times 10^6 / 117.8 = 203.7 \times 10^3$ or $d = 58.8$ say 60 mm **Ans.**

2. Number of turns of the spring coil

Let $\quad n$ = Number of active turns of the spring coil.

We know that the deflection of the spring (δ),

$$250 = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times 160 \times 10^3\ (300)^3\ n}{84 \times 10^3\ (60)^4} = 31.7\ n$$

... (Taking G = 84 MPa = 84×10^3 N/mm^2)

$\therefore \quad n = 250 / 31.7 = 7.88$ say 8 **Ans.**

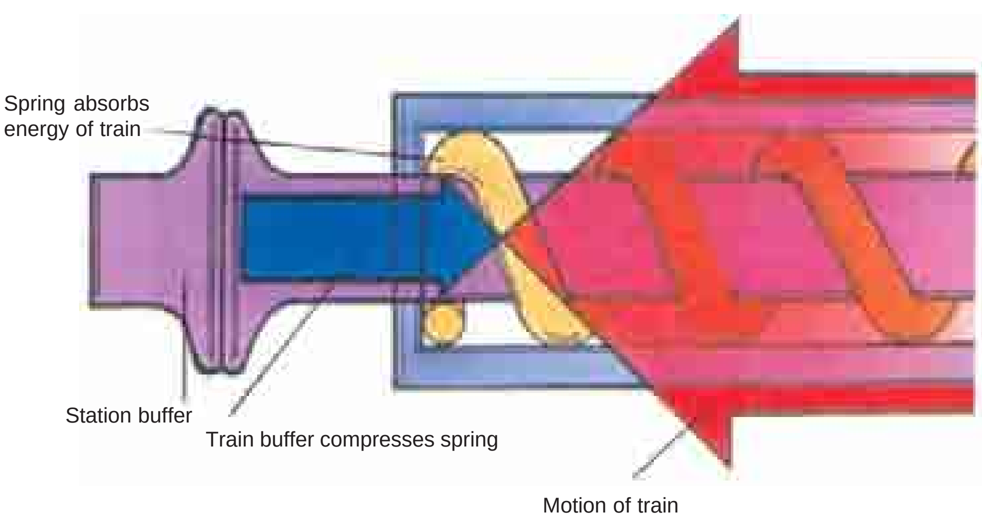
Assuming square and ground ends, total number of turns,

$n' = n + 2 = 8 + 2 = 10$ **Ans.**

3. Free length of the spring

We know that free length of the spring,

$L_F = n'.d + \delta + 0.15\,\delta = 10 \times 60 + 250 + 0.15 \times 250 = 887.5$ mm **Ans.**



4. Pitch of the coil

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{887.5}{10 - 1} = 98.6 \text{ mm} \quad \textbf{Ans.}$$

23.14 Stress and Deflection in Helical Springs of Non-circular Wire

The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space. However these springs have the following main disadvantages :

1. The quality of material used for springs is not so good.
2. The shape of the wire does not remain square or rectangular while forming helix, resulting in trapezoidal cross-sections. It reduces the energy absorbing capacity of the spring.
3. The stress distribution is not as favourable as for circular wires. But this effect is negligible where loading is of static nature.

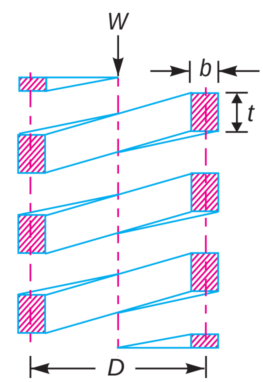


Fig. 23.18. Spring of rectangular wire.

For springs made of rectangular wire, as shown in Fig. 23.18, the maximum shear stress is given by

$$\tau = K \times \frac{W.D\,(1.5\,t + 0.9\,b)}{b^2.t^2}$$

This expression is applicable when the longer side (*i.e.* $t > b$) is parallel to the axis of the spring. But when the shorter side (*i.e.* $t < b$) is parallel to the axis of the spring, then maximum shear stress,

$$\tau = K \times \frac{W.D\,(1.5\,b + 0.9\,t)}{b^2.t^2}$$

and deflection of the spring,

$$\delta = \frac{2.45\,W.D^3.n}{G.b^3\,(t - 0.56\,b)}$$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\,W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568\,W.D^3.n}{G.b^4} = \frac{5.568\,W.C^3.n}{G.b} \qquad \ldots\left(\because C = \frac{D}{b}\right)$$

where b = Side of the square.

Note : In the above expressions,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}, \text{ and } C = \frac{D}{b}$$

Example 23.16. *A loaded narrow-gauge car of mass 1800 kg and moving at a velocity 72 m/min., is brought to rest by a bumper consisting of two helical steel springs of square section. The mean diameter of the coil is six times the side of the square section. In bringing the car to rest, the springs are to be compressed 200 mm. Assuming the allowable shear stress as 365 MPa and spring index of 6, find :*

1. Maximum load on each spring, 2. Side of the square section of the wire, 3. Mean diameter of coils, and 4. Number of active coils.

Take modulus of rigidity as 80 kN/mm^2.

Solution. Given : m = 1800 kg ; v = 72 m/min = 1.2 m/s ; δ = 200 mm ; τ = 365 MPa = 365 N/mm^2 ; C = 6 ; G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

1. Maximum load on each spring,

Let W = Maximum load on each spring.

We know that kinetic energy of the car

$$= \frac{1}{2} m.v^2 = \frac{1}{2} \times 1800\ (1.2)^2 = 1296 \text{ N-m} = 1296 \times 10^3 \text{ N-mm}$$

This energy is absorbed in the two springs when compressed to 200 mm. If the springs are loaded gradually from 0 to W, then

$$\left(\frac{0 + W}{2}\right) 2 \times 200 = 1296 \times 10^3$$

$$\therefore \quad W = 1296 \times 10^3 / 200 = 6480 \text{ N} \quad \textbf{Ans.}$$

2. Side of the square section of the wire

Let b = Side of the square section of the wire, and

D = Mean diameter of the coil = 6 b ... ($\because$ $C = D/b = 6$)

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stres (τ),

$$365 = K \times \frac{2.4\, W.D}{b^3} = 1.2525 \times \frac{2.4 \times 6480 \times 6\, b}{b^3} = \frac{116\ 870}{b^2}$$

$$\therefore \quad b^2 = 116\ 870 / 365 = 320 \quad \text{or} \quad b = 17.89 \text{ say } 18 \text{ mm} \quad \textbf{Ans.}$$

3. Mean diameter of the coil

We know that mean diameter of the coil,

$$D = 6\, b = 6 \times 18 = 108 \text{ mm} \quad \textbf{Ans.}$$

4. Number of active coils

Let n = Number of active coils.

We know that the deflection of the spring (δ),

$$200 = \frac{5.568\, W.C^3.n}{G.b} = \frac{5.568 \times 6480 \times 6^3 \times n}{80 \times 10^3 \times 18} = 5.4\, n$$

$$\therefore \quad n = 200 / 5.4 = 37 \quad \textbf{Ans.}$$

23.15 Helical Springs Subjected to Fatigue Loading

The helical springs subjected to fatigue loading are designed by using the *Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig. 23.19.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to $\tau_e / 2$ and the variable shear stress is also equal to $\tau_e / 2$. A line drawn from A to B (the yield point in shear, τ_y) gives the Soderberg's failure stress line. If a suitable factor of safety ($F.S.$) is applied to the yield strength (τ_y), a safe stress line CD may be drawn parallel to the line AB, as shown in Fig. 23.19. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below :

* We have discussed the Soderberg method for completely reversed stresses in Chapter 6.

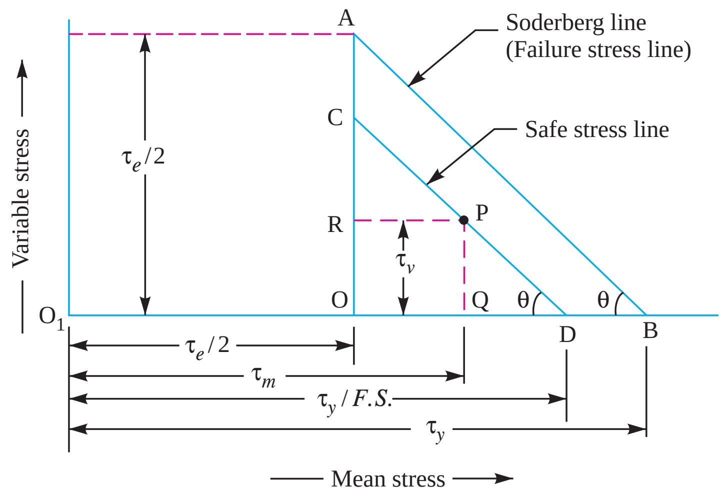


Fig. 23.19. Modified Soderberg method for helical springs.

From similar triangles *PQD* and *AOB*, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\dfrac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e/2}{\tau_y - \dfrac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or $$2\,\tau_v\,.\,\tau_y - \tau_v.\,\tau_e = \frac{\tau_e\,.\,\tau_y}{F.S.} - \tau_m.\tau_e$$

$\therefore$ $$\frac{\tau_e\,.\,\tau_y}{F.S.} = 2\,\tau_v.\tau_y - \tau_v.\tau_e + \tau_m.\tau_e$$

Dividing both sides by $\tau_e.\tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e} \qquad ...(i)$$

Notes : 1. From equation (*i*), the expression for the factor of safety (*F.S.*) may be written as

$$F.S. = \frac{\tau_y}{\tau_m - \tau_v + \dfrac{2\,\tau_v.\tau_y}{\tau_e}}$$

2. The value of mean shear stress (τ_m) is calculated by using the shear stress factor (K_S), while the variable shear stress is calculated by using the full value of the Wahl's factor (*K*). Thus

Mean shear stress,

$$\tau_m = K_s \times \frac{8\,W_m \times D}{\pi d^3}$$

where $$K_S = 1 + \frac{1}{2C}; \text{ and } W_m = \frac{W_{max} + W_{min}}{2}$$

and variable shear stress, $$\tau_v = K \times \frac{8\,W_v \times D}{\pi\, d^3}$$

where $$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}; \text{ and } W_v = \frac{W_{max} - W_{min}}{2}$$

Example 23.17. *A helical compression spring made of oil tempered carbon steel, is subjected to a load which varies from 400 N to 1000 N. The spring index is 6 and the design factor of safety is 1.25. If the yield stress in shear is 770 MPa and endurance stress in shear is 350 MPa, find : 1. Size of the spring wire, 2. Diameters of the spring, 3. Number of turns of the spring, and 4. Free length of the spring.*

The compression of the spring at the maximum load is 30 mm. The modulus of rigidity for the spring material may be taken as 80 kN/mm².

Solution. Given : W_{min} = 400 N ; W_{max} = 1000 N ; C = 6 ; $F.S.$ = 1.25 ; τ_y = 770 MPa = 770 N/mm² ; τ_e = 350 MPa = 350 N/mm² ; δ = 30 mm ; G = 80 kN/mm² = 80 × 10³ N/mm²

1. Size of the spring wire

Let d = Diameter of the spring wire, and

D = Mean diameter of the spring = $C.d = 6\,d$ ... ($\because D/d = C = 6$)

We know that the mean load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{1000 + 400}{2} = 700 \text{ N}$$

and variable load,

$$W_v = \frac{W_{max} - W_{min}}{2} = \frac{1000 - 400}{2} = 300 \text{ N}$$

Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 6} = 1.083$$

Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

We know that mean shear stress,

$$\tau_m = K_S \times \frac{8\,W_m \times D}{\pi\,d^3} = 1.083 \times \frac{8 \times 700 \times 6\,d}{\pi\,d^3} = \frac{11\,582}{d^2} \text{ N/mm}^2$$

and variable shear stress,

$$\tau_v = K \times \frac{8\,W_v \times D}{\pi\,d^3} = 1.2525 \times \frac{8 \times 300 \times 6\,d}{\pi\,d^3} = \frac{5740}{d^2} \text{ N/mm}^2$$

We know that

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\,\tau_v}{\tau_e}$$

$$\frac{1}{1.25} = \frac{\dfrac{11\,582}{d^2} - \dfrac{5740}{d^2}}{770} + \frac{2 \times \dfrac{5740}{d^2}}{350} = \frac{7.6}{d^2} + \frac{32.8}{d^2} = \frac{40.4}{d^2}$$

$\therefore$ $d^2 = 1.25 \times 40.4 = 50.5$ or $d = 7.1$ mm **Ans.**

2. Diameters of the spring

We know that mean diameter of the spring,

$D = C.d = 6 \times 7.1 = 42.6$ mm **Ans.**

Outer diameter of the spring,

$D_o = D + d = 42.6 + 7.1 = 49.7$ mm **Ans.**

and inner diameter of the spring,

$D_i = D - d = 42.6 - 7.1 = 35.5$ mm **Ans.**

3. Number of turns of the spring

Let n = Number of active turns of the spring.

Contents

We know that deflection of the spring (δ),

$$30 = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times 1000\,(42.6)^3\,n}{80 \times 10^3\,(7.1)^4} = 3.04\,n$$

$\therefore \quad n = 30 / 3.04 = 9.87$ say 10 **Ans.**

Assuming the ends of the spring to be squared and ground, the total number of turns of the spring,

$$n' = n + 2 = 10 + 2 = 12 \text{ **Ans.**}$$

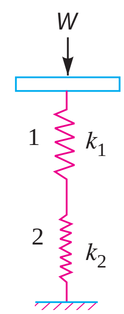
4. Free length of the spring

We know that free length of the spring,

$$L_F = n'.d + \delta + 0.15\,\delta = 12 \times 7.1 + 30 + 0.15 \times 30 \text{ mm}$$
$$= 119.7 \text{ say } 120 \text{ mm}$$ **Ans.**

23.16 Springs in Series

Consider two springs connected in series as shown in Fig. 23.20.



Springs in series.
Fig. 23.20

Let W = Load carried by the springs,

δ_1 = Deflection of spring 1,

δ_2 = Deflection of spring 2,

k_1 = Stiffness of spring 1 = W / δ_1, and

k_2 = Stiffness of spring 2 = W / δ_2

A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

$\therefore$ Total deflection of the springs,

$$\delta = \delta_1 + \delta_2$$

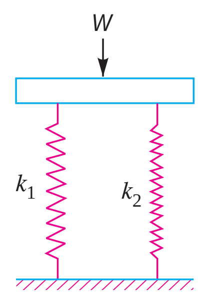
or
$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$\therefore$
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

where k = Combined stiffness of the springs.

23.17 Springs in Parallel

Consider two springs connected in parallel as shown in Fig 23.21.



Springs in parallel.
Fig. 23.21

Let W = Load carried by the springs,

W_1 = Load shared by spring 1,

W_2 = Load shared by spring 2,

k_1 = Stiffness of spring 1, and

k_2 = Stiffness of spring 2.

A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

We know that $W = W_1 + W_2$

or $\delta.k = \delta.k_1 + \delta.k_2$

$\therefore \quad k = k_1 + k_2$

where k = Combined stiffness of the springs, and

δ = Deflection produced.

Top

Example 23.18. *A close coiled helical compression spring of 12 active coils has a spring stiffness of k. It is cut into two springs having 5 and 7 turns. Determine the spring stiffnesses of resulting springs.*

Solution. Given : $n = 12$; $n_1 = 5$; $n_2 = 7$

We know that the deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} \quad \text{or} \quad \frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n}$$

Since G, D and d are constant, therefore substituting

$$\frac{G.d^4}{8\,D^3} = X\text{, a constant, we have } \frac{W}{\delta} = k = \frac{X}{n}$$

or $$X = k.n = 12\,k$$

The spring is cut into two springs with $n_1 = 5$ and $n_2 = 7$.

Let k_1 = Stiffness of spring having 5 turns, and

k_2 = Stiffness of spring having 7 turns.

$$\therefore \quad k_1 = \frac{X}{n_1} = \frac{12\,k}{5} = 2.4\,k \quad \textbf{Ans.}$$

and $$k_2 = \frac{X}{n_2} = \frac{12\,k}{7} = 1.7\,k \quad \textbf{Ans.}$$

23.18 Concentric or Composite Springs

A concentric or composite spring is used for one of the following purposes :

1. To obtain greater spring force within a given space.

2. To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more springs and have the same free lengths as shown in Fig. 23.22 (*a*) and are compressed equally. Such springs are used in automobile clutches, valve springs in aircraft, heavy duty diesel engines and rail-road car suspension systems.

Sometimes concentric springs are used to obtain a spring force which does not increase in a direct relation to the deflection but increases faster. Such springs are made of different lengths as shown in Fig. 23.22 (*b*). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variable centrifugal force.

A car shock absorber.

Contents

The adjacent coils of the concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same stress. In order to get the same stress factor (*K*), it is desirable to have the same spring index (*C*).

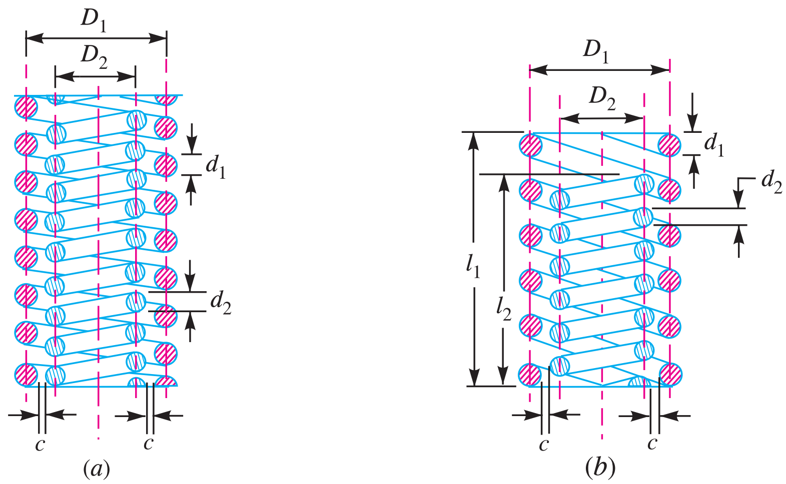


Fig. 23.22. Concentric springs.

Consider a concentric spring as shown in Fig. 23.22 (*a*).

Let W = Axial load,

W_1 = Load shared by outer spring,

W_2 = Load shared by inner spring,

d_1 = Diameter of spring wire of outer spring,

d_2 = Diameter of spring wire of inner spring,

D_1 = Mean diameter of outer spring,

D_2 = Mean diameter of inner spring,

δ_1 = Deflection of outer spring,

δ_2 = Deflection of inner spring,

n_1 = Number of active turns of outer spring, and

n_2 = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stress induced in both the springs is approximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\, W_1 . D_1 . K_1}{\pi\, (d_1)^3} = \frac{8\, W_2 . D_2 . K_2}{\pi\, (d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3} \qquad \text{...}(i)$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

or

$$\frac{8 W_1\, (D_1)^3\, n_1}{(d_1)^4\, G} = \frac{8 W_2\, (D_2)^3\, n_2}{(d_2)^4\, G} \quad \text{or} \quad \frac{W_1\, (D_1)^3\, n_1}{(d_1)^4} = \frac{W_2\, (D_2)^3\, n_2}{(d_2)^4} \qquad \text{...}(ii)$$

Top

When both the springs are compressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e.*

$$n_1.d_1 = n_2.d_2$$

∴ The equation (*ii*) may be written as

$$\frac{W_1 (D_1)^3}{(d_1)^5} = \frac{W_2 (D_2)^3}{(d_2)^5} \quad ...(iii)$$

Now dividing equation (*iii*) by equation (*i*), we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \quad ...(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same.

From equations (*i*) and (*iv*), we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \quad ..(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as $\frac{d_1 - d_2}{2}$.

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or

$$\frac{D_1 - D_2}{2} = d_1 \quad ...(vi)$$

From equation (*iv*), we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation (*vi*), we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$\therefore \quad d_1 (C - 2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2} \quad ...(vii)$$

Example 23.19. *A concentric spring for an aircraft engine valve is to exert a maximum force of 5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solid length and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both the springs is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and (c) the number of active coils in each spring.*

Assume G = 80 kN/mm² and diametral clearance to be equal to the difference between the wire diameters.

Solution. Given : W = 5000 N ; δ = 40 mm ; $\tau_1 = \tau_2$ = 850 MPa = 850 N/mm² ; C = 6 ; G = 80 kN/mm² = 80 × 10³ N/mm²

The concentric spring is shown in Fig. 23.22 (*a*).

(*a*) *Load shared by each spring*

Let W_1 and W_2 = Load shared by outer and inner spring respectively,

d_1 and d_2 = Diameter of spring wires for outer and inner springs respectively, and

D_1 and D_2 = Mean diameter of the outer and inner springs respectively.

* The net clearance between the two springs is given by

$$2\,c = (D_1 - D_2) - (d_1 + d_2)$$

Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

or $$D_1 - D_2 = 2\,d_1$$

We know that $D_1 = C.d_1$, and $D_2 = C.d_2$

$$\therefore \quad C.d_1 - C.d_2 = 2\,d_1$$

or $$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \quad ...(i)$$

We also know that $$\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \quad ...(ii)$$

and $$W_1 + W_2 = W = 5000 \text{ N} \quad ...(iii)$$

From equations (*ii*) and (*iii*), we find that

$$W_1 = 3462 \text{ N, and } W_2 = 1538 \text{ N}$$ **Ans.**

(b) Main dimensions of both the springs

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stress induced in the outer spring (τ_1),

$$850 = K_1 \times \frac{8\,W_1.C}{\pi (d_1)^2} = 1.2525 \times \frac{8 \times 3462 \times 6}{\pi (d_1)^2} = \frac{66\,243}{(d_1)^2}$$

$$\therefore \quad (d_1)^2 = 66\,243 / 850 = 78 \text{ or } d_1 = 8.83 \text{ say } 10 \text{ mm}$$ **Ans.**

and $$D_1 = C.d_1 = 6\,d_1 = 6 \times 10 = 60 \text{ mm}$$ **Ans.**

Similarly, maximum shear stress induced in the inner spring (τ_2),

$$850 = K_2 \times \frac{8W_2.C}{\pi (d_2)^2} = 1.2525 \times \frac{8 \times 1538 \times 6}{\pi (d_2)^2} = \frac{29\,428}{(d_2)^2}$$

$$\therefore \quad (d_2)^2 = 29\,428 / 850 = 34.6 \text{ or } {}^*d_2 = 5.88 \text{ say } 6 \text{ mm}$$ **Ans.**

and $$D_2 = C.d_2 = 6 \times 6 = 36 \text{ mm}$$ **Ans.**

(c) Number of active coils in each spring

Let n_1 and n_2 = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring (δ),

$$40 = \frac{8\,W_1.C^3.n_1}{G.d_1} = \frac{8 \times 3462 \times 6^3 \times n_1}{80 \times 10^3 \times 10} = 7.48\,n_1$$

$$\therefore \quad n_1 = 40 / 7.48 = 5.35 \text{ say } 6$$ **Ans.**

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

∴ Solid length of the outer spring,

$$L_{S1} = n_1' \,.\, d_1 = 8 \times 10 = 80 \text{ mm}$$

Let n_2' be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2'.d_2 = n_1'.d_1$$

* The value of d_2 may also be obtained from equation (*i*), *i.e.*

$$\frac{d_1}{d_2} = 1.5 \quad \text{or} \quad d_2 = \frac{d_1}{1.5} = \frac{8.83}{1.5} = 5.887 \text{ say } 6 \text{ mm}$$

or $$n_2' = \frac{n_1'.d_1}{d_2} = \frac{8 \times 10}{6} = 13.3 \text{ say } 14$$

and $n_2 = 14 - 2 = 12$ **Ans.** ... ($\because n_2' = n_2 + 2$)

Since both the springs have the same free length, therefore

Free length of outer spring

= Free length of inner spring

$= L_{S1} + \delta + 0.15\,\delta = 80 + 40 + 0.15 \times 40 = 126$ mm **Ans.**

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

$= D_1 + d_1 = 60 + 10 = 70$ mm **Ans.**

Inner diameter of the outer spring

$= D_1 - d_1 = 60 - 10 = 50$ mm **Ans.**

Outer diameter of the inner spring

$= D_2 + d_2 = 36 + 6 = 42$ mm **Ans.**

Inner diameter of the inner spring

$= D_2 - d_2 = 36 - 6 = 30$ mm **Ans.**

Shock absorbers

Example 23.20. *A composite spring has two closed coil helical springs as shown in Fig. 23.22 (b). The outer spring is 15 mm larger than the inner spring. The outer spring has 10 coils of mean diameter 40 mm and wire diameter 5mm. The inner spring has 8 coils of mean diameter 30 mm and wire diameter 4 mm. When the spring is subjected to an axial load of 400 N, find 1. compression of each spring, 2. load shared by each spring, and 3. shear stress induced in each spring. The modulus of rigidity may be taken as 84 kN/mm².*

Solution. Given : $\delta_1 = l_1 - l_2 = 15$ mm ; $n_1 = 10$; $D_1 = 40$ mm ; $d_1 = 5$ mm ; $n_2 = 8$; $D_2 = 30$ mm ; $d_2 = 4$ mm ; $W = 400$ N ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. Compression of each spring

Since the outer spring is 15 mm larger than the inner spring, therefore the inner spring will not take any load till the outer spring is compressed by 15 mm. After this, both the springs are compressed together. Let P_1 be the load on the outer spring to compress it by 15 mm.

We know that compression of the spring (δ),

$$15 = \frac{8\,P_1\,(D_1)^3\,n_1}{G\,(d_1)^4} = \frac{8\,P_1\,(40)^3\,10}{84 \times 10^3 \times 5^4} = 0.0975\,P_1$$

$\therefore \quad P_1 = 15 / 0.0975 = 154$ N

Now the remaining load *i.e.* $W - P_1 = 400 - 154 = 246$ N is taken together by both the springs.

Let δ_2 = Further compression of the outer spring or the total compression of the inner spring.

Since for compressing the outer spring by 15 mm, the load required is 154 N, therefore the additional load required by the outer spring to compress it by δ_2 mm is given by

$$P_2 = \frac{P_1}{\delta_1} \times \delta_2 = \frac{154}{15} \times \delta_2 = 10.27\ \delta_2$$

Let W_2 = Load taken by the inner spring to compress it by δ_2 mm.

We know that
$$\delta_2 = \frac{8\ W_2\ (D_2)^3\ n_2}{G\ (d_2)^4} = \frac{8\ W_2\ (30)^3\ 8}{84 \times 10^3 \times 4^4} = 0.08\ W_2$$

$\therefore$
$$W_2 = \delta_2 / 0.08 = 12.5\ \delta_2$$

and
$$P_2 + W_2 = W - P_1 = 400 - 154 = 246 \text{ N}$$

or
$$10.27\ \delta_2 + 12.5\ \delta_2 = 246 \quad \text{or} \quad \delta_2 = 246 / 22.77 = 10.8 \text{ mm}$$ **Ans.**

$\therefore$ Total compression of the outer spring

$$= \delta_1 + \delta_2 = 15 + 10.8 = 25.8 \text{ mm}$$ **Ans.**

2. *Load shared by each spring*

We know that the load shared by the outer spring,

$$W_1 = P_1 + P_2 = 154 + 10.27\ \delta_2 = 154 + 10.27 \times 10.8 = 265 \text{ N}$$ **Ans.**

and load shared by the inner spring,

$$W_2 = 12.5\ \delta_2 = 12.5 \times 10.8 = 135 \text{ N}$$ **Ans.**

Note : The load shared by the inner spring is also given by

$$W_2 = W - W_1 = 400 - 265 = 135 \text{ N}$$ **Ans.**

3. *Shear stress induced in each spring*

We know that the spring index of the outer spring,

$$C_1 = \frac{D_1}{d_1} = \frac{40}{5} = 8$$

and spring index of the inner spring,

$$C_2 = \frac{D_2}{d_2} = \frac{30}{4} = 7.5$$

$\therefore$ Wahl's stress factor for the outer spring,

$$K_1 = \frac{4C_1 - 1}{4C_1 - 4} + \frac{0.615}{C_1} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

and Wahl's stress factor for the inner spring,

$$K_2 = \frac{4C_2 - 1}{4C_2 - 4} + \frac{0.615}{C_2} = \frac{4 \times 7.5 - 1}{4 \times 7.5 - 4} + \frac{0.615}{7.5} = 1.197$$

We know that shear stress induced in the outer spring,

$$\tau_1 = K_1 \times \frac{8\ W_1 . D_1}{\pi\ (d_1)^3} = 1.184 \times \frac{8 \times 265 \times 40}{\pi \times 5^3} = 255.6 \text{ N/mm}^2$$

$$= 255.6 \text{ MPa}$$ **Ans.**

and shear stress induced in the inner spring,

$$\tau_2 = K_2 \times \frac{8\ W_2 . D_2}{\pi\ (d_2)^3} = 1.197 \times \frac{8 \times 135 \times 30}{\pi \times 4^3} = 192.86 \text{ N/mm}^2$$

$$= 192.86 \text{ MPa}$$ **Ans.**

23.19 Helical Torsion Springs

The helical torsion springs as shown in Fig. 23.23, may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc.

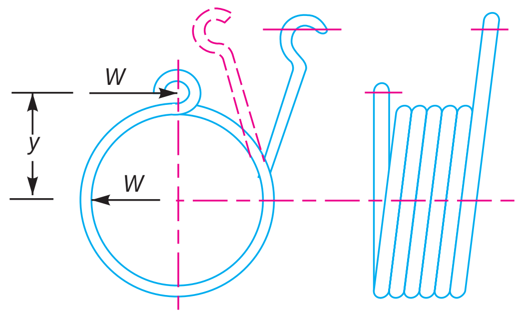


Fig. 23.23. Helical torsion spring.

A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi\,d^3}$$

where K = Wahl's stress factor = $\dfrac{4C^2 - C - 1}{4C^2 - 4C}$,

C = Spring index,

M = Bending moment = $W \times y$,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

and total angle of twist or angular deflection,

$$^*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\,d^4/64} = \frac{64\,M.D.n}{E.d^4}$$

where l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\dfrac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

and deflection, $\delta = \theta \times y = \dfrac{64\,M.D.n}{E.d^4} \times y$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\,M}{t.b^2} = K \times \frac{6\,W \times y}{t.b^2}$$

where $K = \dfrac{3C^2 - C - 0.8}{3C^2 - 3C}$

---

* We know that $M / I = E / R$, where R is the radius of curvature.

$\therefore \quad R = \dfrac{E.I}{M}$ or $\dfrac{l}{\theta} = \dfrac{E.I}{M}$ or $\theta = \dfrac{M.l}{E.I}$ $\quad ...\left(\because R = \dfrac{l}{\theta}\right)$

Angular deflection, $\theta = \dfrac{12\,\pi M.D.n}{E.t.b^3}$; and $\delta = \theta.y = \dfrac{12\,\pi M.D.n}{E.t.b^3} \times y$

In case the spring is made of square wire with each side equal to b, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6\,M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\,\pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\,\pi M.D.n}{E.b^4} \times y$$

Note : Since the diameter of the spring D reduces as the coils wind up under the applied load, therefore a clearance must be provided when the spring wire is to be wound round a mandrel. A small clearance must also be provided between the adjacent coils in order to prevent sliding friction.

Example 23.21. *A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200 kN/mm². The number of effective turns may be taken as 5.5.*

Solution. Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m $= 6000$ N-mm ; $C = 10$; $E = 200$ kN/mm² $= 200 \times 10^3$ N/mm² ; $n = 5.5$

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \quad \textbf{Ans.}$$

23.20 Flat Spiral Spring

A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig. 23.24. These springs are frequently used in watches and gramophones etc.

When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilised in any useful way while the spirals open out slowly. Usually the inner end of spring is clamped to an arbor while the outer end may be pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending.

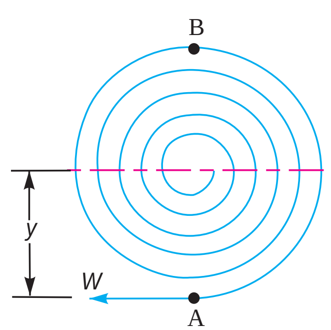


Fig. 23.24. Flat spiral spring.

Let W = Force applied at the outer end A of the spring,

y = Distance of centre of gravity of the spring from A,

l = Length of strip forming the spring,

b = Width of strip,

t = Thickness of strip,

I = Moment of inertia of the spring section = $b.t^3/12$, and

Z = Section modulus of the spring section = $b.t^2/6$

$$\ldots\left(\because\ Z = \frac{I}{y} = \frac{b.t^3}{12 \times t/2} = \frac{b.t^2}{6}\right)$$

When the end A of the spring is pulled up by a force W, then the bending moment on the spring, at a distance y from the line of action of W is given by

$$M = W \times y$$

The greatest bending moment occurs in the spring at B which is at a maximum distance from the application of W.

∴ Bending moment at B,

$$M_B = M_{max} = W \times 2y = 2W.y = 2M$$

∴ Maximum bending stress induced in the spring material,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{2W \times y}{b.t^2/6} = \frac{12W.y}{b.t^2} = \frac{12M}{b.t^2}$$

Flat spiral spring of a mechanical clock.

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by

$$\theta = \frac{M.l}{E.I} = \frac{12\ M.l}{E.b.t^3} \qquad \ldots\left(\because\ I = \frac{b.t^3}{12}\right)$$

and the deflection,

$$\delta = \theta \times y = \frac{M.l.y}{E.I}$$

$$= \frac{12\ M.l.y}{E.b.t^3} = \frac{12W.y^2.l}{E.b.t^3} = \frac{\sigma_b.y.l}{E.t} \qquad \ldots\left(\because\ \sigma_b = \frac{12W.y}{b.t^2}\right)$$

The strain energy stored in the spring

$$= \frac{1}{2}M.\theta = \frac{1}{2}M \times \frac{M.l}{E.I} = \frac{1}{2} \times \frac{M^2.l}{E.I}$$

$$= \frac{1}{2} \times \frac{W^2.y^2.l}{E \times bt^3/12} = \frac{6\ W^2.y^2.l}{E.b.t^3}$$

$$= \frac{6\ W^2.y^2.l}{E.b.t^3} \times \frac{24bt}{24bt} = \frac{144\ W^2y^2}{Eb^2t^4} \times \frac{b\,t\,l}{24}$$

... (Multiplying the numerator and denominator by 24 bt)

$$= \frac{(\sigma_b)^2}{24\ E} \times b\,t\,l = \frac{(\sigma_b)^2}{24\ E} \times \text{Volume of the spring}$$

Example 23.22. *A spiral spring is made of a flat strip 6 mm wide and 0.25 mm thick. The length of the strip is 2.5 metres. Assuming the maximum stress of 800 MPa to occur at the point of greatest bending moment, calculate the bending moment, the number of turns to wind up the spring and the strain energy stored in the spring. Take E = 200 kN/mm².*

Solution. Given : b = 6 mm ; t = 0.25 mm ; l = 2.5 m = 2500 mm ; τ = 800 MPa = 800 N/mm^2 ; E = 200 kN/mm^2 = 200 × 10^3 N/mm^2

Bending moment in the spring

Let M = Bending moment in the spring.

We know that the maximum bending stress in the spring material (σ_b),

$$800 = \frac{12\,M}{b.t^2} = \frac{12\,M}{6\,(0.25)^2} = 32\,M$$

$\therefore$ M = 800 / 32 = 25 N-mm **Ans.**

Number of turns to wind up the spring

We know that the angular deflection of the spring,

$$\theta = \frac{12\,M.l}{E.b.t^3} = \frac{12 \times 25 \times 2500}{200 \times 10^3 \times 6\,(0.25)^3} = 40 \text{ rad}$$

Since one turn of the spring is equal to 2π radians, therefore number of turns to wind up the spring

$= 40 / 2\pi = 6.36$ turns **Ans.**

Strain energy stored in the spring

We know that strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} \times 24 \times 40 = 480 \text{ N-mm}$$ **Ans.**

23.21 Leaf Springs

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.

Consider a single plate fixed at one end and loaded at the other end as shown in Fig. 23.25. This plate may be used as a flat spring.

Let t = Thickness of plate,

b = Width of plate, and

L = Length of plate or distance of the load W from the cantilever end.

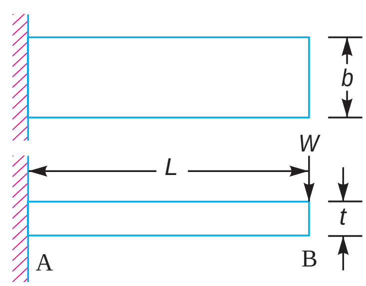


Fig. 23.25. Flat spring (cantilever type).

We know that the maximum bending moment at the cantilever end A,

$$M = W.L$$

and section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

$\therefore$ Bending stress in such a spring,

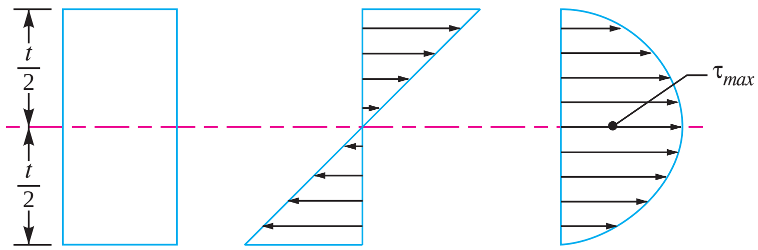
$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2} \qquad \text{...}(i)$$

We know that the maximum deflection for a cantilever with concentrated load at the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3} \qquad \text{...}(ii)$$

$$= \frac{2\,\sigma.L^2}{3\,E.t} \qquad \text{...}\left(\because \sigma = \frac{6W.L}{b.t^2}\right)$$

It may be noted that due to bending moment, top fibres will be in tension and the bottom fibres are in compression, but the shear stress is zero at the extreme fibres and maximum at the centre, as shown in Fig. 23.26. Hence for analysis, both stresses need not to be taken into account simultaneously. We shall consider the bending stress only.



(*a*) Cross-section of plate. (*b*) Bending stress diagram. (*c*) Shear stress diagram.

Fig. 23.26

If the spring is not of cantilever type but it is like a simply supported beam, with length $2L$ and load $2W$ in the centre, as shown in Fig. 23.27, then

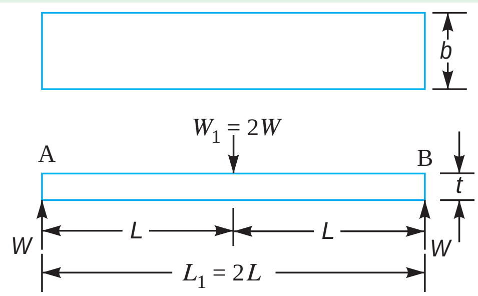


Fig. 23.27. Flat spring (simply supported beam type).

Maximum bending moment in the centre,

$$M = W.L$$

Section modulus, $Z = b.t^2 / 6$

$\therefore$ Bending stress, $\sigma = \dfrac{M}{Z} = \dfrac{W.L}{b.t^2/6}$

$$= \frac{6\,W.L}{b.t^2}$$

We know that maximum deflection of a simply supported beam loaded in the centre is given by

Leaf spring

$$\delta = \frac{W_1\,(L_1)^3}{48\,E.I} = \frac{(2W)\,(2L)^3}{48\,E.I} = \frac{W.L^3}{3\,E.I}$$

...($\because$ In this case, $W_1 = 2W$, and $L_1 = 2L$)

Contents

From above we see that a spring such as automobile spring (semi-elliptical spring) with length $2L$ and loaded in the centre by a load $2W$, may be treated as a double cantilever.

If the plate of cantilever is cut into a series of n strips of width b and these are placed as shown in Fig. 23.28, then equations (*i*) and (*ii*) may be written as

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \text{...}(iii)$$

and

$$\delta = \frac{4\,W.L^3}{n.E.b.t^3} = \frac{2\,\sigma.L^2}{3\,E.t} \qquad \text{...}(iv)$$

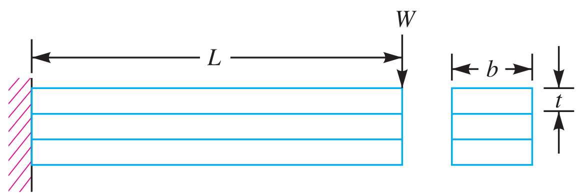


Fig. 23.28

The above relations give the stress and deflection of a leaf spring of uniform cross-section. The stress at such a spring is maximum at the support.

If a triangular plate is used as shown in Fig. 23.29 (*a*), the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. 23.29 (*b*) to form a graduated or laminated leaf spring, then

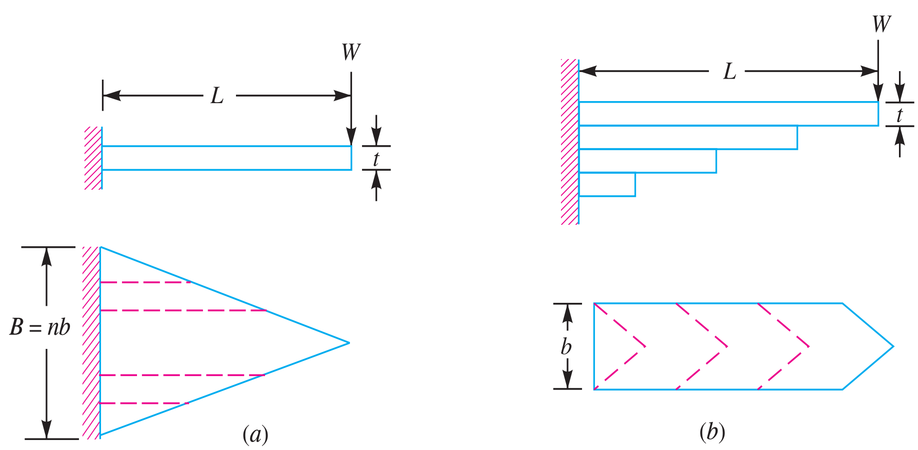


Fig. 23.29. Laminated leaf spring.

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \text{...}(v)$$

and

$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad \text{...}(vi)$$

where n = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

Top

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from equations **(*iv*)** and **(*vi*)** that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes $_F$ and $_G$ are used to indicate the full length (or uniform cross-section) and graduated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6W_F.L}{n_F.b.t^2} = \frac{3}{2}\left[\frac{6\,W_G.L}{n_G.b.t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2}\times\frac{W_G}{n_G}$$

$$\therefore \quad \frac{W_F}{W_G} = \frac{3\,n_F}{2\,n_G} \qquad \text{...(vii)}$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G}+1 = \frac{3\,n_F}{2\,n_G}+1 \quad \text{or} \quad \frac{W_F+W_G}{W_G} = \frac{3\,n_F+2\,n_G}{2\,n_G}$$

$$\therefore \quad W_G = \left(\frac{2\,n_G}{3\,n_F+2\,n_G}\right)(W_F+W_G) = \left(\frac{2\,n_G}{3\,n_F+2\,n_G}\right)W \qquad \text{...(viii)}$$

where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

From equation **(*vii*)**, we may write

$$\frac{W_G}{W_F} = \frac{2\,n_G}{3\,n_F}$$

or

$$\frac{W_G}{W_F}+1 = \frac{2\,n_G}{3\,n_F}+1 \qquad \text{... (Adding 1 to both sides)}$$

$$\frac{W_G+W_F}{W_F} = \frac{2\,n_G+3\,n_F}{3\,n_F}$$

$$\therefore \quad W_F = \left(\frac{3\,n_F}{2\,n_G+3\,n_F}\right)(W_G+W_F) = \left(\frac{3\,n_F}{2\,n_G+3\,n_F}\right)W \qquad \text{...(ix)}$$

$\therefore$ Bending stress for full length leaves,

$$\sigma_F = \frac{6\,W_F.L}{n_F.b\,t^2} = \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G+3\,n_F}\right)W = \frac{18\,W.L}{b.t^2\,(2\,n_G+3\,n_F)}$$

Since $\sigma_F = \frac{3}{2}\sigma_G$, therefore

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3}\times\frac{18\,W.L}{b.t^2\,(2\,n_G+3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G+3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation **(*iv*)**, *i.e.*

$$\delta = \frac{2\,\sigma_F\times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G+3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G+3\,n_F)}$$

23.22 Construction of Leaf Spring

A leaf spring commonly used in automobiles is of semi-elliptical form as shown in Fig. 23.30.

It is built up of a number of plates (known as leaves). The leaves are usually given an initial curvature or cambered so that they will tend to straighten under the load. The leaves are held together by means of a band shrunk around them at the centre or by a bolt passing through the centre. Since the band exerts a stiffening and strengthening effect, therefore the effective length of the spring for bending will be overall length of the spring *minus* width of band. In case of a centre bolt, two-third distance between centres of *U*-bolt should be subtracted from the overall length of the spring in order to find effective length. The spring is clamped to the axle housing by means of *U*-bolts.

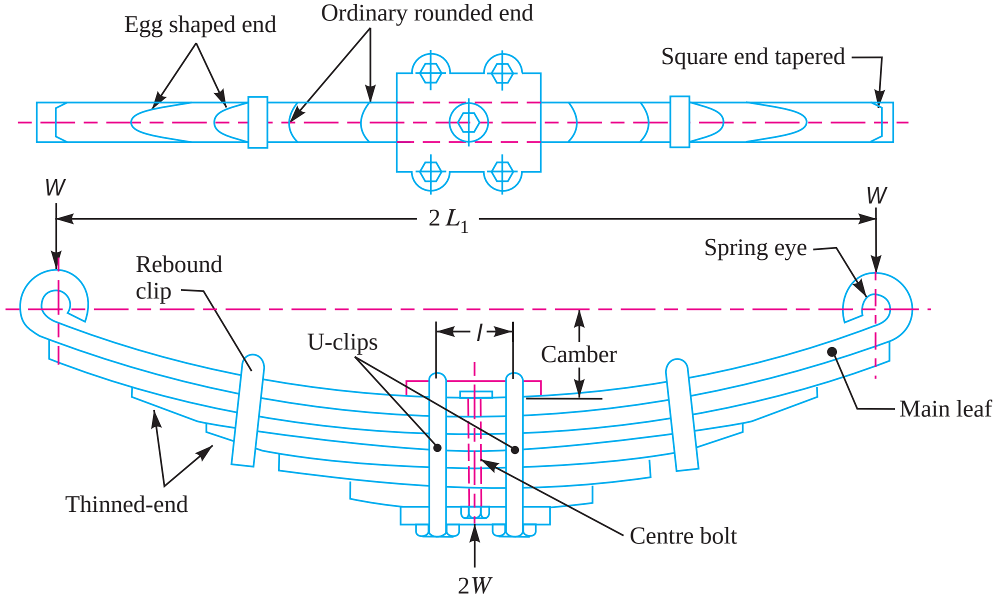


Fig. 23.30. Semi-elliptical leaf spring.

The longest leaf known as ***main leaf*** or ***master leaf*** has its ends formed in the shape of an eye through which the bolts are passed to secure the spring to its supports. Usually the eyes, through which the spring is attached to the hanger or shackle, are provided with bushings of some antifriction material such as bronze or rubber. The other leaves of the spring are known as ***graduated leaves.*** In order to prevent digging in the adjacent leaves, the ends of the graduated leaves are trimmed in various forms as shown in Fig. 23.30. Since the master leaf has to with stand vertical bending loads as well as loads due to sideways of the vehicle and twisting, therefore due to the presence of stresses caused by these loads, it is usual to provide two full length leaves and the rest graduated leaves as shown in Fig. 23.30.

Rebound clips are located at intermediate positions in the length of the spring, so that the graduated leaves also share the stresses induced in the full length leaves when the spring rebounds.

Leaf spring fatigue testing system.

23.23 Equalised Stress in Spring Leaves (Nipping)

We have already discussed that the stress in the full length leaves is 50% greater than the stress in the graduated leaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed. This condition may be obtained in the following two ways :

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small distance from the neutral axis to the edge of the leaf.

2. By giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. 23.31, before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by *C* in Fig. 23.31, is called ***nip.*** When the central bolt, holding the various leaves together, is tightened, the full length leaf will bend back as shown dotted in Fig. 23.31 and have an initial stress in a direction opposite to that of the normal load. The graduated

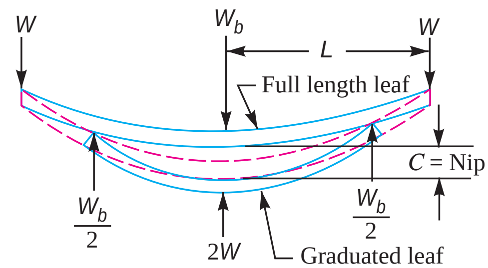


Fig. 23.31

leaves will have an initial stress in the same direction as that of the normal load. When the load is gradually applied to the spring, the full length leaf is first relieved of this initial stress and then stressed in opposite direction. Consequently, the full length leaf will be stressed less than the graduated leaf. The initial gap between the leaves may be adjusted so that under maximum load condition the stress in all the leaves is equal, or if desired, the full length leaves may have the lower stress. This is desirable in automobile springs in which full length leaves are designed for lower stress because the full length leaves carry additional loads caused by the swaying of the car, twisting and in some cases due to driving the car through the rear springs. Let us now find the value of initial gap or nip *C*.

Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap *C*. In other words,

$$\delta_G = \delta_F + C$$

$$\therefore \qquad C = \delta_G - \delta_F = \frac{6\,W_G\,.\,L^3}{n_G\,E.b.t^3} - \frac{4\,W_F.L^3}{n_F.E.b.t^3} \qquad ...(i)$$

Since the stresses are equal, therefore

$$\sigma_G = \sigma_F$$

$$\frac{6\,W_G.L}{n_G.b.t^2} = \frac{6\,W_F.L}{n_F.b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} = \frac{W_F}{n_F}$$

$$\therefore \qquad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

and

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of W_G and W_F in equation (*i*), we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad ...(ii)$$

The load on the clip bolts (W_b) required to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \qquad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\,L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

or

$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\,W_b}{2\,n_G} = \frac{2\,n_G.W_b + 3\,n_F.W_b}{2\,n_F.n_G} = \frac{W_b\,(2\,n_G + 3\,n_F)}{2\,n_F.n_G}$$

$$\therefore \qquad W_b = \frac{2\,n_F.n_G.W}{n\,(2\,n_G + 3\,n_F)} \qquad ...(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

$$\therefore \text{ Final stress, } \qquad \sigma = \frac{6\,W_F.L}{n_F.b.t^2} - \frac{6\,L}{n_F.b.t^2} \times \frac{W_b}{2} = \frac{6\,L}{n_F.b.t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\,L}{n_F.b.t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F.n_G.W}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6W.L}{b.t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3n - n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3(n_F + n_G) - n_G}{n\,(2n_G + 3n_F)}\right] = \frac{6\,W.L}{n.b.t^2} \qquad ...(iv)$$

... (Substituting $n = n_F + n_G$)

Notes : 1. The final stress in the leaves is also equal to the stress in graduated leaves due to the applied load *plus* the initial stress.

2. The deflection in the spring due to the applied load is same as without initial stress.

23.24 Length of Leaf Spring Leaves

The length of the leaf spring leaves may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

l = Width of band or distance between centres of *U*-bolts. It is the ineffective length of the spring,

n_F = Number of full length leaves,

n_G = Number of graduated leaves, and

n = Total number of leaves = $n_F + n_G$.

We have already discussed that the effective length of the spring,

$$2L = 2L_1 - l \qquad ...(\text{When band is used})$$

$$= 2L_1 - \frac{2}{3}\,l \qquad ...(\text{When } U\text{-bolts are used})$$

It may be noted that when there is only one full length leaf (*i.e.* master leaf only), then the number of leaves to be cut will be n and when there are two full length leaves (including one master leaf), then the number of leaves to be cut will be $(n - 1)$. If a leaf spring has two full length leaves, then the length of leaves is obtained as follows :

$$\text{Length of smallest leaf} = \frac{\text{Effective length}}{n-1} + \text{Ineffective length}$$

$$\text{Length of next leaf} = \frac{\text{Effective length}}{n-1} \times 2 + \text{Ineffective length}$$

Similarly, length of $(n-1)$th leaf

$$= \frac{\text{Effective length}}{n-1} \times (n-1) + \text{Ineffective length}$$

The nth leaf will be the master leaf and it is of full length. Since the master leaf has eyes on both sides, therefore

$$\text{Length of master leaf} = 2\,L_1 + \pi\,(d + t) \times 2$$

where d = Inside diameter of eye, and

t = Thickness of master leaf.

The approximate relation between the radius of curvature (R) and the camber (y) of the spring is given by

$$R = \frac{(L_1)^2}{2y}$$

The exact relation is given by

$$y\,(2R + y) = (L_1)^2$$

where L_1 = Half span of the spring.

Note : The maximum deflection (δ) of the spring is equal to camber (y) of the spring.

23.25 Standard Sizes of Automobile Suspension Springs

Following are the standard sizes for the automobile suspension springs:

1. Standard nominal widths are : 32, 40*, 45, 50*, 55, 60*, 65, 70*, 75, 80, 90, 100 and 125 mm. (Dimensions marked* are the preferred widths)
2. Standard nominal thicknesses are : 3.2, 4.5, 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11, 12, 14 and 16 mm.
3. At the eye, the following bore diameters are recommended : 19, 20, 22, 23, 25, 27, 28, 30, 32, 35, 38, 50 and 55 mm.
4. Dimensions for the centre bolts, if employed, shall be as given in the following table.

Table 23.5. Dimensions for centre bolts.

Width of leaves in mm	*Dia. of centre bolt in mm*	*Dia. of head in mm*	*Length of bolt head in mm*
Upto and including 65	8 or 10	12 or 15	10 or 11
Above 65	12 or 16	17 or 20	11

5. Minimum clip sections and the corresponding sizes of rivets and bolts used with the clips shall be as given in the following table (See Fig. 23.32).

Table 23.6. Dimensions of clip, rivet and bolts.

Spring width (B) in mm	*Clip section ($b \times t$) in mm × mm*	*Dia. of rivet (d_1) in mm*	*Dia. of bolt (d_2) in mm*
Under 50	20 × 4	6	6
50, 55 and 60	25 × 5	8	8
65, 70, 75 and 80	25 × 6	10	8
90, 100 and 125	32 × 6	10	10

Top

Contents

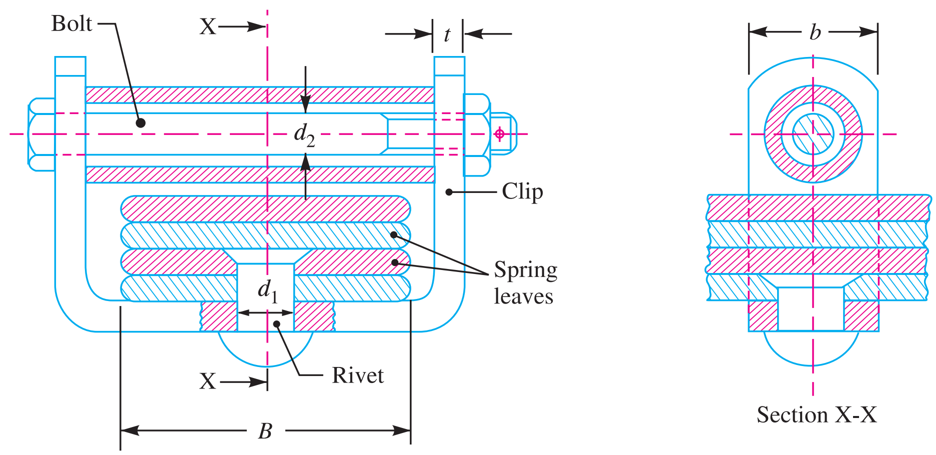


Fig. 23.32. Spring clip.

Notes : 1. For springs of width below 65 mm, one rivet of 6, 8 or 10 mm may be used. For springs of width above 65 mm, two rivets of 6 or 8 mm or one rivet of 10 mm may be used.

2. For further details, the following Indian Standards may be referred :

(*a*) IS : 9484 – 1980 (Reaffirmed 1990) on 'Specification for centre bolts for leaf springs'.

(*b*) IS : 9574 – 1989 (Reaffirmed 1994) on 'Leaf springs assembly-Clips-Specification'.

23.26 Materials for Leaf Springs

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon. The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.

According to Indian standards, the recommended materials are :

1. For automobiles : 50 Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.
2. For rail road springs : C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (water-hardened) and 55 Si 2 Mn 90 (oil-hardened).
3. The physical properties of some of these materials are given in the following table. All values are for oil quenched condition and for single heat only.

Table 23.7. Physical properties of materials commonly used for leaf springs.

Material	*Condition*	*Ultimate tensile strength (MPa)*	*Tensile yield strength (MPa)*	*Brinell hardness number*
50 Cr 1	Hardened	1680 – 2200	1540 – 1750	461 – 601
50 Cr 1 V 23	and	1900 – 2200	1680 – 1890	534 – 601
55 Si 2 Mn 90	tempered	1820 – 2060	1680 – 1920	534 – 601

Note : For further details, Indian Standard [IS : 3431 – 1982 (Reaffirmed 1992)] on 'Specification for steel for the manufacture of volute, helical and laminated springs for automotive suspension' may be referred.

Example 23.23. *Design a leaf spring for the following specifications :*

Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm.

Take Young's modulus, E = 200 kN/mm^2 and allowable stress in spring material as 600 MPa.

Top

Solution. Given : Total load = 140 kN ; No. of springs = 4; n = 10 ; $2L$ = 1000 mm or L = 500 mm ; δ = 80 mm ; E = 200 kN/mm² = 200 × 10³ N/mm² ; σ = 600 MPa = 600 N/mm²

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$$\therefore \quad W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$$\therefore \quad n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3 \quad \text{...(i)}$$

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$$\therefore \quad n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6 \quad \text{...(ii)}$$

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm} \quad \textbf{Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm} \quad \textbf{Ans.}$$

Example 23.24. *A truck spring has 12 number of leaves, two of which are full length leaves. The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness and width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.*

Solution. Given : n = 12 ; n_F = 2 ; $2L_1$ = 1.05 m = 1050 mm ; l = 85 mm ; $2W$ = 5.4 kN = 5400 N or W = 2700 N ; σ_F = 280 MPa = 280 N/mm²

Thickness and width of the spring leaves

Let t = Thickness of the leaves, and

b = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring (b) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$$\therefore \quad L = 965 / 2 = 482.5 \text{ mm}$$

Contents

and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves (σ_F),

$$280 = \frac{18\, W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\, t \times t^2\ (2 \times 10 + 3 \times 2)} = \frac{225\ 476}{t^3}$$

$\therefore$ $t^3 = 225\ 476 / 280 = 805.3$ or $t = 9.3$ say 10 mm **Ans.**

and $b = 4\, t = 4 \times 10 = 40$ mm **Ans.**

Deflection of the spring

We know that deflection of the spring,

$$\delta = \frac{12\, W.L^3}{E.b.t^3\ (2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\ (2 \times 10 + 3 \times 2)} \text{ mm}$$

$= 16.7$ mm **Ans.** ... (Taking $E = 210 \times 10^3$ N/mm^2)

Example 23.25. *A locomotive semi-elliptical laminated spring has an overall length of 1 m and sustains a load of 70 kN at its centre. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100 mm width. All the leaves are to be stressed to 400 MPa, when fully loaded. The ratio of the total spring depth to that of width is 2. E = 210 kN/mm^2. Determine :*

1. The thickness and width of the leaves.

2. The initial gap that should be provided between the full length and graduated leaves before the band load is applied.

3. The load exerted on the band after the spring is assembled.

Solution. Given : $2L_1 = 1$ m $= 1000$ mm ; $2W = 70$ kN or $W = 35$ kN $= 35 \times 10^3$ N ; $n_F = 3$; $n_G = 15$; $l = 100$ mm ; $\sigma = 400$ MPa $= 400$ N/mm^2 ; $E = 210$ kN/mm^2 $= 210 \times 10^3$ N/mm^2

1. Thickness and width of leaves

Let t = Thickness of leaves, and

b = Width of leaves.

We know that the total number of leaves,

$$n = n_F + n_G = 3 + 15 = 18$$

Since it is given that ratio of the total spring depth ($n \times t$) and width of leaves is 2, therefore

$$\frac{n \times t}{b} = 2 \quad \text{or} \quad b = n \times t / 2 = 18 \times t / 2 = 9\, t$$

We know that the effective length of the leaves,

$$2L = 2L_1 - l = 1000 - 100 = 900 \text{ mm} \quad \text{or} \quad L = 900 / 2 = 450 \text{ mm}$$

Since all the leaves are equally stressed, therefore final stress (σ),

$$400 = \frac{6\, W.L}{n.b.t^2} = \frac{6 \times 35 \times 10^3 \times 450}{18 \times 9\, t \times t^2} = \frac{583 \times 10^3}{t^3}$$

$\therefore$ $t^3 = 583 \times 10^3 / 400 = 1458$ or $t = 11.34$ say 12 mm **Ans.**

and $b = 9\, t = 9 \times 12 = 108$ mm **Ans.**

Top

2. *Initial gap*

We know that the initial gap (C) that should be provided between the full length and graduated leaves before the band load is applied, is given by

$$C = \frac{2\,W.L^3}{n.E.b.t^3} = \frac{2 \times 35 \times 10^3\ (450)^3}{18 \times 210 \times 10^3 \times 108\ (12)^3} = 9.04 \text{ mm} \quad \textbf{Ans.}$$

3. *Load exerted on the band after the spring is assembled*

We know that the load exerted on the band after the spring is assembled,

$$W_b = \frac{2\,n_F.n_G.W}{n\,(2n_G + 3n_F)} = \frac{2 \times 3 \times 15 \times 35 \times 10^3}{18\ (2 \times 15 + 3 \times 3)} = 4487 \text{ N} \quad \textbf{Ans.}$$

Example 23.26. *A semi-elliptical laminated vehicle spring to carry a load of 6000 N is to consist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. The spring is to be 1.1 m in length and attached to the axle by two U-bolts 80 mm apart. The bolts hold the central portion of the spring so rigidly that they may be considered equivalent to a band having a width equal to the distance between the bolts. Assume a design stress for spring material as 350 MPa. Determine :*

1. Thickness of leaves, 2. Deflection of spring, 3. Diameter of eye, 4. Length of leaves, and 5. Radius to which leaves should be initially bent.

Sketch the semi-elliptical leaf-spring arrangement.

The standard thickness of leaves are : 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11 etc. in mm.

Solution. Given : $2W = 6000$ N or $W = 3000$ N ; $n = 7$; $b = 65$ mm ; $n_F = 2$; $2L_1 = 1.1$ m $= 1100$ mm or $L_1 = 550$ mm ; $l = 80$ mm ; $\sigma = 350$ MPa $= 350$ N/mm²

1. *Thickness of leaves*

Let t = Thickness of leaves.

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1100 - 80 = 1020 \text{ mm}$$

$$\therefore \quad L = 1020 / 2 = 510 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the maximum stress (σ_F),

$$350 = \frac{18\,W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 3000 \times 510}{65 \times t^2\ (2 \times 5 + 3 \times 2)} = \frac{26\,480}{t^2} \quad \ldots (\sigma_F = \sigma)$$

$$\therefore \quad t^2 = 26\,480 / 350 = 75.66 \quad \text{or} \quad t = 8.7 \text{ say } 9 \text{ mm} \quad \textbf{Ans.}$$

2. *Deflection of spring*

We know that deflection of spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\ (2n_G + 3n_F)} = \frac{12 \times 3000\ (510)^3}{210 \times 10^3 \times 65 \times 9^3\ (2 \times 5 + 3 \times 2)}$$

$$= 30 \text{ mm } \textbf{Ans.} \quad \ldots (\text{Taking } E = 210 \times 10^3 \text{ N/mm}^2)$$

3. *Diameter of eye*

The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let d = Inner diameter of the eye or diameter of the pin,

l_1 = Length of the pin which is equal to the width of the eye or leaf (*i.e.* b) = 65 mm ...(Given)

p_b = Bearing pressure on the pin which may be taken as 8 N/mm².

We know that the load on pin (W),

$$3000 = d \times l_1 \times p_b$$
$$= d \times 65 \times 8 = 520\ d$$
$$\therefore \quad d = 3000 / 520$$
$$= 5.77 \text{ say } 6 \text{ mm}$$

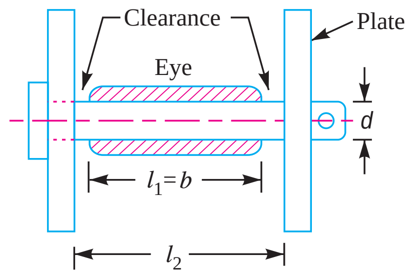


Fig. 23.33

Let us now consider the bending of the pin. Since there is a clearance of about 2 mm between the shackle (or plate) and eye as shown in Fig. 23.33, therefore length of the pin under bending,

$$l_2 = l_1 + 2 \times 2 = 65 + 4 = 69 \text{ mm}$$

Leaf spring.

Maximum bending moment on the pin,

$$M = \frac{W \times l_2}{4} = \frac{3000 \times 69}{4} = 51\ 750 \text{ N-mm}$$

and section modulus, $$Z = \frac{\pi}{32} \times d^3 = 0.0982\ d^3$$

We know that bending stress (σ_b),

$$80 = \frac{M}{Z} = \frac{51\ 750}{0.0982\ d^3} = \frac{527 \times 10^3}{d^3} \qquad \text{... (Taking } \sigma_b = 80 \text{ N/mm}^2)$$

$$\therefore \quad d^3 = 527 \times 10^3 / 80 = 6587 \quad \text{or} \quad d = 18.7 \text{ say } 20 \text{ mm}$$ **Ans.**

We shall take the inner diameter of eye or diameter of pin (d) as 20 mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin (W),

$$3000 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2\ \tau = 628.4\ \tau$$

$$\therefore \quad \tau = 3000 / 628.4 = 4.77 \text{ N/mm}^2, \text{ which is safe.}$$

4. Length of leaves

We know that ineffective length of the spring

$= l = 80$ mm ... ($\because$ U-bolts are considered equivalent to a band)

$\therefore$ Length of the smallest leaf $= \dfrac{\text{Effective length}}{n-1} + \text{Ineffective length}$

$$= \frac{1020}{7-1} + 80 = 250 \text{ mm} \quad \textbf{Ans.}$$

Length of the 2nd leaf $= \dfrac{1020}{7-1} \times 2 + 80 = 420$ mm **Ans.**

Length of the 3rd leaf $= \dfrac{1020}{7-1} \times 3 + 80 = 590$ mm **Ans.**

Length of the 4th leaf $= \dfrac{1020}{7-1} \times 4 + 80 = 760$ mm **Ans.**

Length of the 5th leaf $= \dfrac{1020}{7-1} \times 5 + 80 = 930$ mm **Ans.**

Length of the 6th leaf $= \dfrac{1020}{7-1} \times 6 + 80 = 1100$ mm **Ans.**

The 6th and 7th leaves are full length leaves and the 7th leaf (*i.e.* the top leaf) will act as a master leaf.

We know that length of the master leaf

$$= 2L_1 + \pi (d + t)\, 2 = 1100 + \pi (20 + 9)2 = 1282.2 \text{ mm} \quad \textbf{Ans.}$$

5. *Radius to which the leaves should be initially bent*

Let R = Radius to which the leaves should be initially bent, and

y = Camber of the spring.

We know that

$$y\,(2R - y) = (L_1)^2$$

$$30(2R - 30) = (550)^2 \quad \text{or} \quad 2R - 30 = (550)^2/30 = 10\,083 \qquad \text{... } (\because y = \delta)$$

$$\therefore \quad R = \frac{10\,083 + 30}{2} = 5056.5 \text{ mm} \quad \textbf{Ans.}$$

EXERCISES

1. Design a compression helical spring to carry a load of 500 N with a deflection of 25 mm. The spring index may be taken as 8. Assume the following values for the spring material:

Permissible shear stress = 350 MPa

Modulus of rigidity = 84 kN/mm^2

Wahl's factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, where C = spring index.

[Ans. d = 5.893 mm ; D = 47.144 mm ; n = 6]

2. A helical valve spring is to be designed for an operating load range of approximately 90 to 135 N. The deflection of the spring for the load range is 7.5 mm. Assume a spring index of 10. Permissible shear stress for the material of the spring = 480 MPa and its modulus of rigidity = 80 kN/mm^2. Design the spring.

Take Wahl's factor $= \dfrac{4C-1}{4C-4} + \dfrac{0.615}{C}$, C being the spring index.

[Ans. d = 2.74 mm ; D = 27.4 mm ; n = 6]